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STANDARD - 11

PHYSICS

BOARD / NEET / JEE

PART-01 | CHAPTER- 08 | GRAVITATION

GYANMANJARI CAREER ACADEMY

As per NCERT Syllabus

STANDARD - 11
PART - I

PHYSICS

FOR GUJ. BOARD, NEET, JEE

: Developed By :

GYANMANJARI CAREER ACADEMY

B - 6300, KALIYABID, BHAVNAGAR - 364002
CONTACT : +91-70690 20211

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કોપીરાઈટ કાયદાની કલમ ૬૩ હેઠળ પુસ્તકો કોપી રાઈટેડ છે. અમારા પુસ્તકની ઝેરોક્ષ કરવી કે કરાવવી એ બંને ગંભીર ગુનો છે. આ પુસ્તકનાં લખાણ, શૈલી, ગોઠવણી તથા રજુઆતનો બીજા કોઈ પણ પુસ્તક, મેગેઝિન, ગાઈડ માટે કે બીજી કોઈ પણ રીતે ઉપયોગ કરતાં પહેલાં પ્રકાશક તથા લેખકોની લેખિત મંજૂરી મેળવવી ખાસ જરૂરી છે.

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Chapter - 08 - Gravitation

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STANDARD-11 CH - 8 : GRAVITATION

THEORY

1 Introduction

→ The stars in the sky and the planets revolving around the sun have been attracting the attention of the scientists since ancient time.

→ First scientific study of the solar system was carried out by the Greeks. The principle of Greek astronomy proposed by Ptolemy, nearly 2000 years ago, is known as **geo-centric theory.**

- According to this theory the Earth is stationary at the centre of the universe and all celestial bodies - stars, sun, planets all of them-are revolving around the earth.
- Ptolemy proposed their motions to be circular. According to him the planets move on circular paths and the centres of those circles move on larger circles.
- But Aryabhatt in the fifth century, proposed a theory that all palnets revolve on the circles with the sun at the centre.

→ Then, almost one thousand years later Nicolaus Copernicus (1473-1543) of Poland proposed a definitive model about the planets revolving on perfect circles with the sun at the center. This is known as **helio-centric theory.**

- Thus it was support to the theory of Aryabhatt. Copernicus model was not accepted by the recognised institutions of that time. But Galileo supported his theory.

→ Tyco Brahe (1546-1601) of Denmark had accumulated many observations, about planetary motion by direct eye, during his life-time. These observations were studied by Johannes Kepler (1571-1640) who gave three laws of planetary motion. They are known as Kepler's laws. In this chapter we will study these laws, Newton's Law of Gravitation and the satellites.

2 Kepler's Laws :

→ From the study of the observations recorded by Tycho Brathe, Johannes Kelper gave three laws of planetary motion. They are called Kepler's laws. They are as follows :

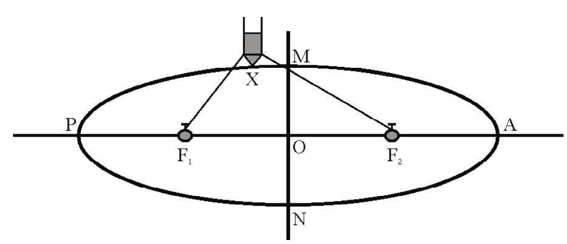
2.1 First Law (Law of Orbits) :

→ **"All the planets move in the elliptical orbits with the sun situated at on of the foci."**

[Only for information :

An ellipse can be drawn as under :

Keep the ends of a string of length l fixed at points F_1 and F_2, Where $F_1F_2 < l$. Now, keep the tip of a pencil with the string and move it such that the string remains tight. The curve PNAM obtained in this way is an ellipse as in figure.



OP = a = OA

OM = b = ON

Here, $F_1X + F_2X$ = constant. It shows the charesteristic of an ellipse.

Moreover, if a = b, the elipse becomes a circle.]

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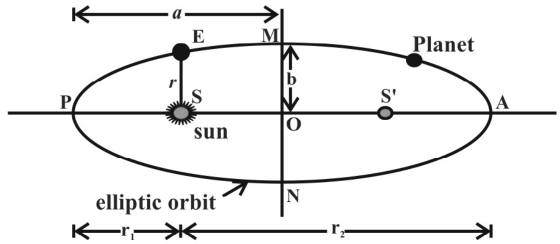


Ellipitc orbit of the planet

If r_1 and r_2 are the shortest and the longest distances of planet from the sun respectively then $(2/r)=(1/r_1)+(1/r_2)$. where, r is the distance between planet and sun, when the planet is perpendicular to the major axis. **[Solved problem-1]**

→ PA = 2a, MN = 2b
OP = OA = a = semi-major axis

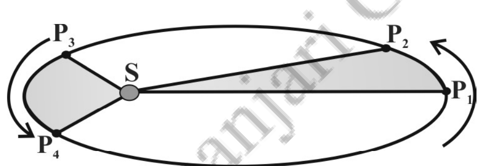
→ In above figure the ellipse PNAM showing the path of a planet has two foci S and S'.

- This law of orbit suggests different shapes from the circular orbits suggested by Copernicus.

2.2 Second Law (Law of Areas) :

→ **"The line joining the Sun and the planet sweeps equal areas in equal intervals of time."** (see following figure)

Area covered per unit time = Arial velocity = (dA / dt) = (L / 2m) = constant. (from the conservation of angular momentum) This low is the geometrical representation of conservation of angular momentum. $\vec{L}=\vec{r}\times\vec{p}$



Areal velocity is constant

→ When the planet is away from the sun, it goes from P_1 to P_2 in certain time-interval Δt and when it is near the sun it goes from P_3 to P_4 the same time-interval.
Hence, according to this law,
area of SP_1P_2 = area of SP_3P_4 .

→ This law has been obtained from the observation that a planet moves slower in the orbit when it is far away from the sun and it moves faster when it is near to the sun.

→ We can call the area swept in unit time as areal velocity (= area/time) and this law indicates that the areal velocity is constant. You have already seen this aspect in Chapter-10.

2.3 Third Law (Law of Periods) :

→ **"The square of the time-period (T) of the revolution of a planet is proportional to the cube of the semi-major axis (a) of its elliptical orbit." That is, $T^2 \propto a^3$.**

**From $T \propto a^{3/2}$,
If $a' = 4a$, then
$T' = 8T$.**

- The **time-period (T)** means the time required to complet one revolution.(★)

3 Newton's Universal Law of Gravitation :

3.1 Newton's universal law of gravitation is as follows :
"Every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them."

- The direction of this force is along the line joining them.
- This force is called the **gravitational force.**

→ According to this law, the magnitude of the force on the **particle 1** of mass m_1, by the other particle 2 of mass m_2, lying at distance r from it.

By changing medium gravitational force remains as it is.

→ is $$F_{12} = \frac{G\, m_1\, m_2}{r^2} \quad (1)$$

- The direction of this force is from particle 1 to the particle 2 (in the direction of $\vec{r}_{12}$), see following figure.

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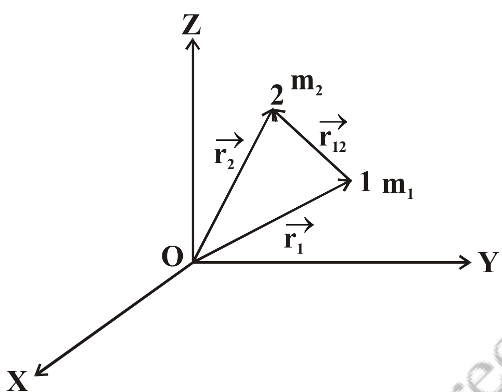
- Here, G is a constant and it is called the universal constant of gravitation, because its value is the same at all places at all times in the whole of the universe.
- The value of G was first determined by Cavendish experimentally. Thereafter many scientists also have determined its value more precisely.
- At present the accepted value of G is 6.67×10^{-11} N m^2 / kg^2.
- The dimensional formula for G is $M^{-1} L^3 T^{-2}$.

3.2 In order to write the equation (1) in the vector form consider following figure.



→ From the figure,

$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$

$$\hat{r}_{12} = \frac{\vec{r}_{12}}{|\vec{r}_{12}|} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_{12}|}$$

$$\hat{r}_{12} = \frac{\vec{r}_2 - \vec{r}_1}{r} \quad \text{——(2)}$$

→ Here, $r = |\vec{r}_{12}|$

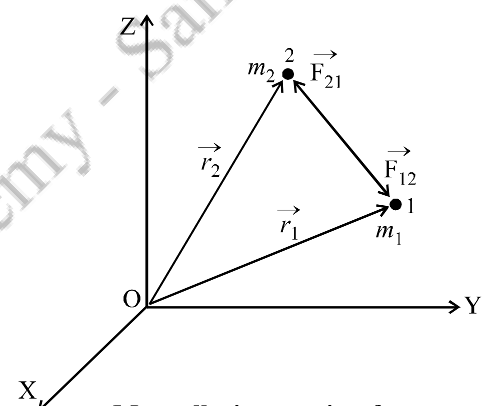
→ It is clear from the figure, that

$$\begin{pmatrix} \vec{F}_{12} \\ \text{force on} \\ 1 \text{ by } 2 \end{pmatrix} = \frac{G\, m_1\, m_2}{r^2}\, \hat{r}_{12} \quad \text{——(3)}$$

→ Since the gravitational foces are mutually interactive forces, the force exerted on particle 1 by particle 2, ($\vec{F}_{12}$) is the same and in opposite direction to the force exerted on particle 2 by particle 1, ($\vec{F}_{21}$).

$$\begin{pmatrix} \vec{F}_{21} \\ \text{force on} \\ 2 \text{ by } 1 \end{pmatrix} = \frac{-G\, m_1\, m_2}{r^2}\, \hat{r}_{12} = \frac{G\, m_1\, m_2}{r^2}\, \hat{r}_{21} \quad \text{——(4)}$$

→ Both these forces $\vec{F}_{12}$ and $\vec{F}_{21}$ are shown in following figure.



Mutually interactive force

3.3 **Force due to an extended object :**

→ An extended object can be considered as a collection of point masses. (i.e. particles) The force due to such an extended object **on a point mass** is equal to vector sum of the forces exerted on it by all the poit masses in the extended object. Thus the force on particle 1 by an extentended object is,

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \ldots.. \quad \text{——(1)}$$

$$= \frac{Gm_1m_2}{r^2_{12}}\hat{r}_{12} + \frac{Gm_1m_3}{r^2_{13}}\hat{r}_{13} + \frac{Gm_1m_4}{r^2_{14}}\hat{r}_{14} + \ldots + \frac{Gm_1m_n}{r^2_{1n}}\hat{r}_{1n} \quad \text{(2)}$$

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→ In the same way we can find the total force **on an extended object by another extended object** by the vector sum of the forces on every point mass of one object by every mass of the other object. This can be done easily with the help of calculus.

3.4 Special Cases :

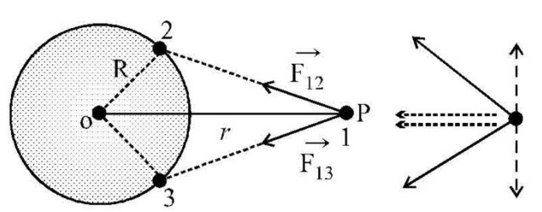
(1) The gravitational force by a hollow spherical shell of uniform density on a particle outside and on the surface of the shell is equal to the force wich can be obtained by considering the entire mass of shell as concentrated on its center. (★)

It is also true for solid sphere.

(2) The force on a particle at any point inside a hollow spherical shell of uniform density is zero.(✦)

★ **Qualititative explanation - only for information :**
The forces on the particle 1 by the particles 2 and 3 on the shell are $\vec{F}_{12}$ and $\vec{F}_{13}$. Consider their components (i) parallel to OP and (ii) perpendicular to OP. The components perpendicular to OP are cancelled and the components parallel to OP are added. Such a process can be thought for the particles on symmentric positions with respect to line OP on the shell. Thus, it can be seen that the resultant force is towards the centre. We shall accept without giving proof that its magnitude is obtained as mentioned above.

✦ **Qualititative explanation - only for information :**
Different particles of the shell attract the given particle in different directions and the resultant of those forces becomes zero. This also we will accept without giving proof.



For r > R, the force due to the shell is towards the centre of the shell

Solved Problems

(1) The largest and the shortest distance of the earth from the sun are r_1 and r_2. Its distance from the sun when it is perpendicular to the major axis of the orbit drawn from the sun ....

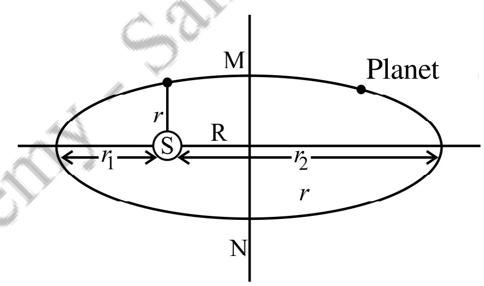
[NEET : 1988]

(A) $\frac{r_1 + r_2}{4}$ (B) $\frac{r_1 + r_2}{r_1 - r_2}$

(C) $\frac{2r_1 r_2}{r_1 + r_2}$ (D) $\frac{r_1 + r_2}{3}$

Solution : **[Ans. : C]**

→ From Kepler's first law,



→ If r_1 and r_2 are the shortest and longest distance of planet from the sun respectively then

$$\frac{2}{r} = \frac{1}{r_1} + \frac{1}{r_2}$$

where r is the distance between planet and sun, when the planet is perpendicular to the major axis.

$$\therefore \quad \boxed{r = \frac{2r_1 r_2}{r_1 + r_2}}$$

(2) Let the speed of the planet at the perihelion P in Figure be v_P and the Sun-planet distance SP be r_P. Relate $[r_P, v_P]$ to the corresponding quantities at the aphelion $[r_A, v_A]$. Will the planet take equal times to traverse BAC and CPB ?

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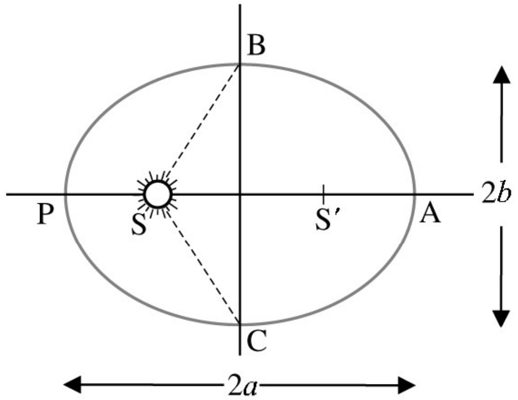

Solution :

→ The magnitude of the angular momentum at P is $L_p = m_p r_p v_p$

→ Since inspection tells us that r_p and v_p are mutually perpendicular.

Similarly, $L_A = m_p r_A v_A$.

→ From angular momentum conservation

$m_p v_p r_p = m_p v_A r_A$ or

$$\frac{v_p}{v_A} = \frac{r_A}{r_P}$$

→ Since $r_A > r_p$, $v_p > v_A$.

→ The area SBAC bounded by the ellipse and the radius vectors SB and SC is larger than SBPC in figure.

→ From Kepler's second law, equal areas are swept in equal times.

→ Hence the planet will take a longer time to traverse BAC than CPB.

(3) A planet moving along an elliptical orbit is closest to the sun at a distance r_1 and farthest away at a distance of r_2. If v_1 and v_2 are the linear velocities at these points respectively then the ratio $\frac{v_1}{v_2}$ = ... [NEET : 2011]

(A) $\left(\frac{r_2}{r_1}\right)^2$ **(B)** $\frac{r_2}{r_1}$

(C) $\frac{r_1}{r_2}$ **(D)** $\left(\frac{r_1}{r_2}\right)^2$

Solution : [Ans. : B]

→ From conservation of angular momentum, $L_1 = L_2$

$\therefore \quad mv_1r_1 = mv_2r_2$

$$\Rightarrow \frac{v_1}{v_2} = \frac{r_2}{r_1}$$

(4) The period of revolution of planet A round the sun is 8 times that of B. The distance of A from the sun is how many times greater than that of B from the sun ? [NEET : 1997]

(A) 5 (B) 4 (C) 3 (D) 2

Solution : [Ans. : B]

→ According to Kepler's third law,

$T^2 \propto r^3$

$$\therefore \frac{T_A^{\ 2}}{T_B^{\ 2}} = \frac{r_A^{\ 3}}{r_B^{\ 3}}$$

$$\therefore \frac{r_A}{r_B} = \left(\frac{T_A}{T_B}\right)^{2/3} = (8)^{2/3}$$

$= 4$

$r_A = 4\, r_B$

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(5) The position vector of the objects of masses 25 kg and 10 kg are (4,7,5)m and (1,3,5)m respectively. Obtain the vector representing the gravitational force on 25 kg object by 10 kg object. (Take $G = 6.67 \times 10^{-11}$ Nm^2/kg^2)

Solution :

→ Here, $m_1 = 25$ kg,

$m_2 = 10$ kg,

$\vec{r}_1 = (4,7,5)$m,

$\vec{r}_2 = (1,3,5)$m

$\vec{F}_{12} = ?$

$$\vec{F}_{12} = \frac{Gm_1m_2}{r^2}\hat{r}_{12} \quad \text{———} (1)$$

[Force on 1 by 2]

→ $\vec{r}_{12} = \vec{r}_2 - \vec{r}_1 = (1,3,5) - (4,7,5) = (-3,-4,0)$m

$\therefore\ r = |\vec{r}_{12}| = \sqrt{(-3)^2 + (-4)^2 + (0)^2} = 5$m

→ and $\hat{r}_{12} = \frac{\vec{r}_{12}}{|\vec{r}_{12}|} = \frac{(-3,-4,0)}{5} = (-0.6, -0.8, 0)$ m

→ Substituting these values in equation (1)

$$\vec{F}_{12} = (6.67 \times 10^{-11}) \frac{(25 \times 10)}{5^2} (-0.6, -0.8, 0)$$

$$= (6.67 \times 10^{-10}) (-0.6\,\hat{i} - 0.8\,\hat{j})\ N$$

(6) A space craft goes from the Earth directly to the sun. How far from the centre of the Earth the gravitational forces exerted on it by the Earth and by the sun would be of equal magnitude ? The distance between the Earth and the sun is 1.49×10^8 km. The masses of the sun and the Earth are 2×10^{30} kg and 6×10^{24} kg respectively.

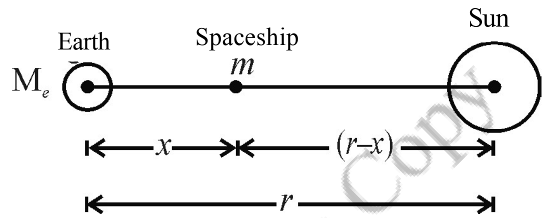
Solution :

→ Let the spaceship of mass m be at 'x' distance from the Earth value $F_e = F_s$

→ $r = 1.49 \times 10^8$ km $= 1.49 \times 10^{11}$ m

$M_e = 6 \times 10^{24}$ kg

$M_S = 2 \times 10^{30}$ kg



→ Gravitational force applied on space craft by the Earth = gravitational force by the sun on the space craft.

$$\frac{GM_em}{x^2} = \frac{GM_Sm}{(r-x)^2}$$

$$\frac{(r-x)^2}{x^2} = \frac{M_S}{M_e} = \frac{2 \times 10^{30}}{6 \times 10^{24}}$$

$$\frac{r-x}{x} = \frac{10^3}{\sqrt{3}}$$

$$\sqrt{3}r - \sqrt{3}x = 1000\,x$$

$$(1000 + \sqrt{3})\,x = \sqrt{3}\,r$$

$$x = \frac{\sqrt{3}\,r}{1000 + \sqrt{3}}$$

$$= \frac{\sqrt{3}\,(1.49 \times 10^{11})}{1000 + \sqrt{3}}$$

$$\boxed{x = 2.57 \times 10^5\ km}$$

(7) The mass of one object is M. How should the body be divided into two parts, so that the force acting between the two parts is maximum for a given separation between them ?

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Solution :

→ The condition for the force to become maximum $\frac{dF}{dm} = 0$

→ Suppose the mass of one piece is m and another is (M − m) and the distance between them is r.

$$F = \frac{Gm(M-m)}{r^2}$$

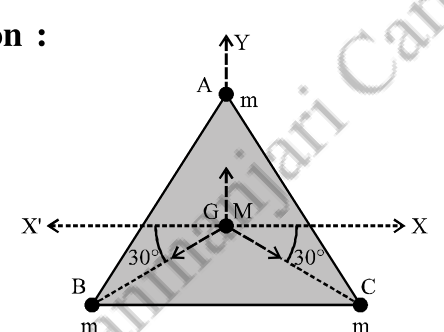
$$\frac{dF}{dm} = \frac{G}{r^2}\frac{d}{dm}[mM - m^2]$$

$$0 = \frac{G}{r^2}[M - 2m]$$

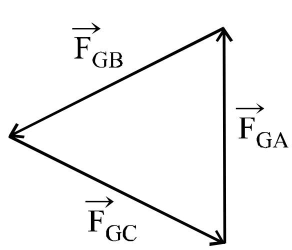
$$\therefore \quad m = \frac{M}{2}$$

(8) At each vertex of an equilateral triangle a particle of mass m kg is kept. What is the gravitational force acting on a mass M kg placed at centroid of the triangle ? The distance of centroid from the vertex is 1 m.

Solution :



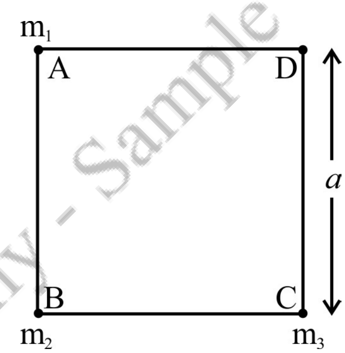
→ From given figure, sum of all three force vectors $\vec{F}_{GA}$, $\vec{F}_{GB}$ and $\vec{F}_{GC}$ as per the triangular method is given below.



→ Resultant force of three forces becomes zero because here vectors denoting force makes closed loop.

(9) Three masses of equal value are placed at the three vertices of a square. If the force acts between m_1 and m_2 is F_{12} and between m_1 and m_3 is F_{13} , then find $\frac{F_{12}}{F_{13}}$.

Solution :



→ Suppose length of side of square is a, from pythagoras theorem,

$$AC^2 = AD^2 + CD^2$$

$$= a^2 + a^2$$

$$AC = \sqrt{2}\,a$$

→ $F_{12} = \frac{Gm^2}{a^2}$

→ $F_{13} = \frac{Gm^2}{(\sqrt{2}\,a)^2}$

⇒ $\frac{F_{12}}{F_{13}} = 2$

(10) At each vertex of a square a particle of mass m kg is kept. The length of each side of a square is *a*. Then find the gravitational force acting on any one mass placed at any vertex.

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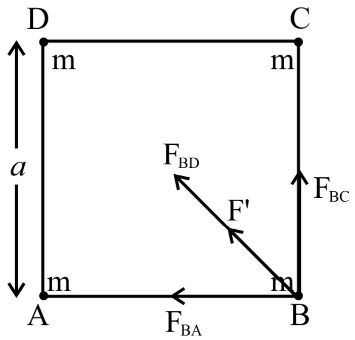
Solution :



→ Suppose we want to calculate resultant force on mass m placed at vertex B, due to all masses.

→ From figure,

→ $F_{BA} = F_{BC} = \frac{Gm^2}{a^2}$ and

$F' = \frac{\sqrt{2}\ Gm^2}{a^2}$ and

$F_{BD} = \frac{Gm^2}{2a^2}$

→ Resultant force,

→ $F_B = F_{BD} + F'$

$= \frac{Gm^2}{2\ a^2} + \frac{\sqrt{2}\ Gm^2}{a^2}$

$F_B = \frac{Gm^2}{a^2} \left(\frac{1}{2} + \sqrt{2} \right)$

(11) Four particles each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is ....

[JEE-2014]

(A) $\sqrt{\frac{GM}{R}(1 + 2\sqrt{2})}$

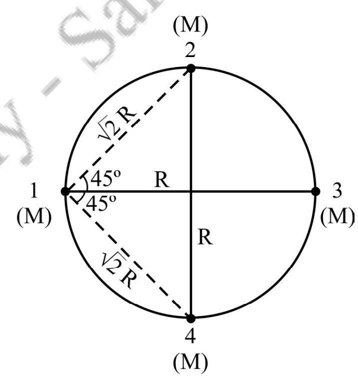
(B) $\frac{1}{2}\sqrt{\frac{GM}{R}(1 + 2\sqrt{2})}$

(C) $\sqrt{\frac{GM}{R}}$

(D) $\sqrt{2\sqrt{2}\frac{GM}{R}}$

Solution : **[Ans. : B]**

→ Here, resultant gravitational force on any one particle due to all is equal and its direction is towards to centre of circular path.



→ From figure resultant gravitaional force F on particle 1 is,

→ $F = F_{12}\ \cos 45^\circ + F_{13} + F_{14}\ \cos 45^\circ$

$= \frac{Gm^2}{(\sqrt{2}\ R)^2} \times \frac{1}{\sqrt{2}} + \frac{Gm^2}{(2\ R)^2} + \frac{Gm^2}{(\sqrt{2}\ R)^2} \times \frac{1}{\sqrt{2}}$

$= 2\left(\frac{Gm^2}{2R^2}\right) \times \frac{1}{\sqrt{2}} + \frac{Gm^2}{4\ R^2}$

$= \frac{Gm^2}{2R^2} \left(\frac{1}{\sqrt{2}} + \frac{1}{4} \right)$

$\therefore \frac{Mv^2}{R} = \frac{Gm^2}{R^2} \left(\frac{4 + \sqrt{2}}{4\sqrt{2}} \right)$

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$$v^2 = \frac{Gm}{R}\left(\frac{4+\sqrt{2}}{4\sqrt{2}}\right)$$

$$v = \frac{1}{2}\sqrt{\frac{Gm}{R}}\sqrt{2\sqrt{2}+1}$$

$$v = \frac{1}{2}\sqrt{\frac{Gm}{R}\ (1+2\sqrt{2})}$$

(12) Choose any one of the following four responses :

(A) If both Assertion and Reason are true and reason is the correct explanation of the Assertion.

(B) If both Assertion and Reason are true but Reason is not a correct explanation of the Assertion.

(C) If Assertion is true but Reason is false

(D) If both Assertion and Reason are false.

Assertion : Moon has no atmosphere.

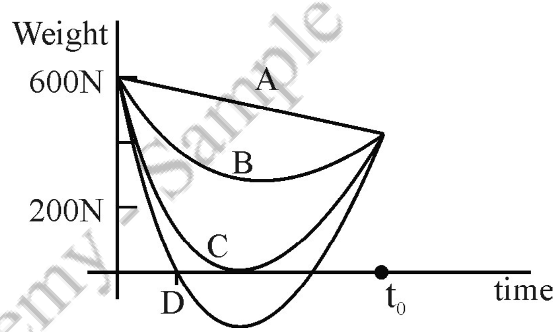
Reason : If the gas-molecules are formed on the surface of moon, the average kinetic energy of the molecule at the temperature prevailing there, is just equal to their potential energy.

(A) A (B) B

(C) C (D) D

Solution : [Ans. : C]

(13) Suppose, the acceleration due to gravity at the Earth's surface is 10 m/s^2 and at the surface of Mars it is 4.0 m/s^2. A 60 kg pasenger goes from the Earth to the Mars in a spaceship moving with a constant velocity. Neglect all other objects in the sky. Which part of figure best represents the weight (net gravitational force) of the passenger as a function of time ? [AIIMS : 2012]



(A) A (B) B (C) C (D) D

Solution : [Ans. : C]

NOTE

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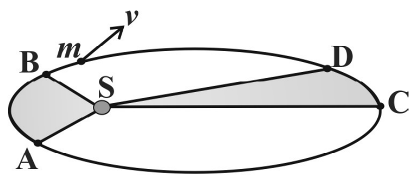
Current Topic Practice

(14) The figure shows elliptical orbit of a planet m about the sun S. The shaded area SCD is twice the shaded area SAB. If t_1 is the time for the planet to move from C to D and t_2 is the time to move from A to B, then . [NEET : 2009]



Areal velocity is constant

(A) $t_1 > t_2$ (B) $t_1 = 2t_2$

(C) $t_1 = 4t_2$ (D) $t_1 = t_2$

(15) Make suitable pairs :

I	II
(P) Kepler's 1st law	(1) Law of periods
(*Q*) Kepler's 2nd law	(2) Law of orbits
	(3) Law of areas

(A) P → 1, *Q* → 2 (B) P → 2, *Q* → 3

(C) P → 3, *Q* → 2 (D) P → 2, *Q* → 1

(16) Kepler's second law about the constancy of the areal velocity of the planet is the consequence of the law of conservation of

(A) energy

(B) linear momentum

(C) angular momentum

(D) charge

(17) When two bodies are in air, the gravitational force between them is F_1. If they are put inside water at the same separation, what would be the force between them ?

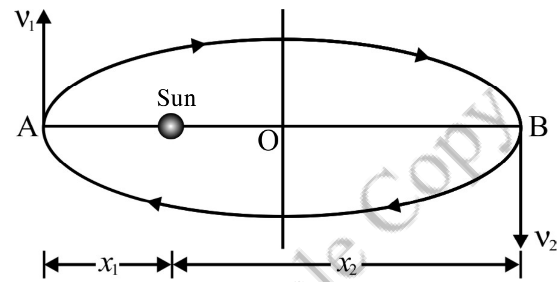
(A) Same as F_1

(B) $F_1 / 2$

(C) Any value less then F_1

(D) 2 F_1

(18) In the figure if the speed of the planet at A is v_1; what is the speed at B ?



(A) $v_2 = v_1x_1 / x_2$ (B) $v_2 = v_1x_2 / x_1$

(C) $v_2 = x_2 / v_1x_1$ (D) $v_2 = x_1 / v_1x_2$

(19) Which of the following is the dimensional formula for G ?

(A) $M^1L^3T^{-2}$ (B) $M^{-1}L^{-3}T^2$

(C) $M^{-1}L^3T^{-2}$ (D) $M^{-1}L^{-3}T^{-2}$

(20) Two spheres of masses m and M are situated in air and the gravitational force between them is F. The space around the masses is now filled with a liquid of specific density 3. The gravitational force will now be......... [NEET : 2003]

(A) F / 3 (B) F / 9

(C) 3F (D) F

(21) Two particles of mass m are revolving on a circle due to gravitational force what is the speed of each particle about a centre of mass. [JEE : 2011]

(A) $\sqrt{\dfrac{Gm}{R}}$ (B) $\sqrt{\dfrac{Gm}{4R}}$

(C) $\sqrt{\dfrac{Gm}{3R}}$ (D) $\sqrt{\dfrac{Gm}{2R}}$

Answer : (14) B (15) B (16) C (17) A (18) A (19) C (20) D (21) B

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(22) At each vertex of a square a particle of mass m kg is kept. What is the gravitational force acting on a mass M kg placed at the intersection point of two diagonal. The distance of intersection of two diagonal from the vertex is 1 m.

(23) At each vertex of an equilateral triangle a particle of mass 2 kg is kept. What is the gravitational force acting on a mass 4 kg placed at the centroid of the triangle ? The distance of centroid from the vertex is 2 m.

(24) If the mass of moon is (M/81), where M is the mass of earth, find the distance of the point from the moon, where gravitational field due to earth and moon cancel each other. Given that distance between earth and moon is 60 R where R is the radius of earth : [AIIMS : 2000]

(A) 4 R (B) 8 R

(C) 2 R (D) 6 R

(25) The radius of the circular orbit of the Earth, revolving around the sun is 1.5×10^8 km. The orbital speed of the Earth is 30 km/s. Calculate the mass of the sun from this data. $G = 6.67 \times 10^{-11}$ Nm^2/kg^2.

(26) Which of the following has the unit $N\ m^2 / kg^2$?

(A) linear momentum

(B) gravitational force

(C) universal constant of gravitation

(D) gravitational acceleration.

(27) For a planet revolving around the sun

(A) linear speed and angular speed are constant

(B) areal velocity and angular momentum are constant

(C) linear speed and areal velocity are constant

(D) areal velocity is constant but angular momentum changes

(28) A planet is moving in an elliptical orbit around the sun. If T, U, E and L stand for its kinetic energy, gravitational potential energy, total energy and magnitude of angular momentum about the centre of force, which of the following is correct ? [NEET : 1990]

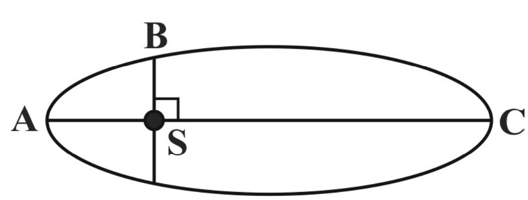
(A) T is conserved

(B) U is always positive

(C) E is always negative

(D) L is conserved but direction of vector L changes continuously

(29) The K.E. of a planet in an elliptical orbit about the Sun, at positions A , B and C are K_A , K_B and K_C , respectively. AC is major axis and SB is perpendicular to AC at the position of the Sun S as shown in the figure. Then, [NEET : 2018]



(A) $K_B > K_A > K_C$ (B) $K_A < K_B < K_C$

(C) $K_B < K_A < K_C$ (D) $K_A > K_B > K_C$

Answer : (24) D (26) C (27) B (28) C (29) D

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(30) Kepler's third law states that square of period of revolution (T) of a planet around the sun, is proportional to third power of average distance r between sun and planet. i.e. $T^2 = Kr^3$, here K is constant. If the masses of sun and planet are M and m respectively then as per Newton's law of gravitation force of attraction between them is, $F = (GMm / r^2)$, here G is gravitational constant. The relation between G and K is described as : [NEET : 2015]

(A) $GK = 4\pi^2$ (B) $GMK = 4\pi^2$
(C) $K = G$ (D) $K = 1 / G$

(31) The force of gravitation is : [AIIMS : 2007]

(A) repulsive
(B) conservative
(C) electrostatic
(D) non-conservative

(32) A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth. Then [AIIMS : 2011]

(A) the acceleration of S is always directed towards the centre of the earth
(B) the angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant
(C) the total mechanical energy of S varies periodically with time
(D) the linear momentum of S remains constant in magnitude

(33) A comet orbits the sun in a highly elliptical orbit. Does the comet have a constant

(a) linear speed,
(b) angular speed,
(c) angular momentum,
(d) kinetic energy,
(e) potential energy,
(f) total energy throughout its orbit ?

Neglect any mass loss of the comet when it comes very close to the Sun.

NOTE

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Answer : (30) B (31) B (32) A

Hints & Solution

(14)

→ Applying Kepler's second law,

∴ $\frac{dA}{dt}$ = constant

∴ $\frac{A_1}{t_1} = \frac{A_2}{t_2}$

→ Here A_1 = area under SCD

A_2 = area under ABS

∴ $t_1 = \frac{A_1}{A_2} t_2$

$A_1 = 2 A_2$

∴ $t_1 = 2 t_2$

(17)

→ Gravitational field is independent of the medium.

(18)

→ From conservation of angular momentum, $L_1 = L_2$

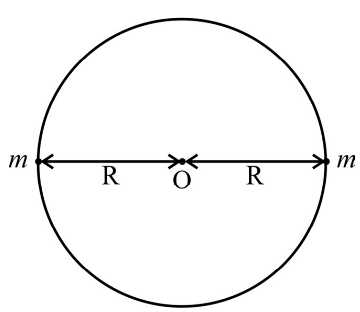
∴ $mv_1x_1 = mv_2x_2$

∴ $v_2 = \frac{v_1x_1}{x_2}$

(20)

→ Gravitational force is independent from the medium.

(21)

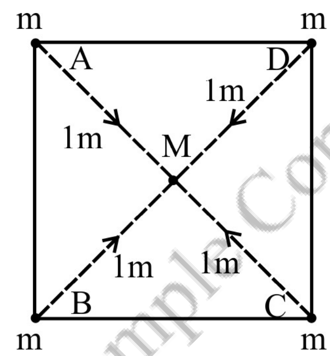


→ Centripital force = gravitational force

$$\frac{mv^2}{R} = \frac{Gm^2}{(2R)^2}$$

∴ $v = \sqrt{\frac{Gm^2}{4R}}$

(22)



→ Suppose the intersection point of two of diagonal is O and gravitaional force on mass M due to all masses is,

$$\vec{F}_{OA} = \frac{GMm}{1^2} (\vec{AO})$$

$$\vec{F}_{OC} = \frac{GmM}{1^2} (\vec{CO})$$

$$\vec{F}_{OB} = \frac{GMm}{1^2} (\vec{BO})$$

$$\vec{F}_{OD} = \frac{GMm}{1^2} (\vec{DO})$$

→ Here all the forces are equal and opposite to each other net resultant force at point O, $\vec{F}_O = 0$.

(24)

→ As per Solved problem No. 6.

(25)

→ r $= 1.5\times 10^8$ *km* $= 1.5\times 10^{11}$ *m*

v = 30 *km/sec* = 3×10^4 *m/s*

M_S = ?

G $= 6.67 \times 10^{-11}$ Nm^2/kg^2

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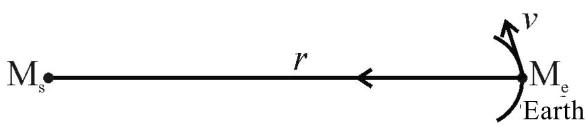
→ when earth rotates around the sun with the velocity v

→ centrepetal force on the earth = Gravitational force by the sun



$$\frac{M_e v^2}{r} = \frac{GM_eM_S}{r^2}$$

$$M_S = \frac{r v^2}{G}$$

$$= \frac{(1.5\times 10^{11})(3\times 10^4)^2}{6.67\times 10^{-11}}$$

$$\boxed{M_S = 2.02\times 10^{30}\ kg}$$

(26)

→ $F = \dfrac{G m_1 m_2}{r^2}$ $\quad\therefore\ G = \dfrac{Fr^2}{m_1 m_2}$

$\therefore$ unit of G = unit of $\dfrac{Fr^2}{m_1 m_2}$ = Nm^2 / kg^2

(27)

→ From the law of conservation of angular momentum...

(29)

→ At position A, potential energy is minimum,

$\left(\because\ U = -\dfrac{GMm}{r}\right)$

$\therefore$ So kinetic energy is maximum.

→ At position C, U is maximum.

$\therefore$ So kinetic energy is minimum,

$K_A > K_B > K_C$

(30)

→ We know that,

$$T^2 = \frac{4\pi^2 r^3}{GM} \qquad \text{... ... ...(1)}$$

$$T^2 = Kr^3 \qquad \text{... ... ...(2)}$$

→ From equation (1) and (2)

$GMK = 4\pi^2$

(33)

→ A comet while going on elliptical orbit around the sun has constant angular momentum and total energy at all locations, but other quantities vary with locations.

NOTE

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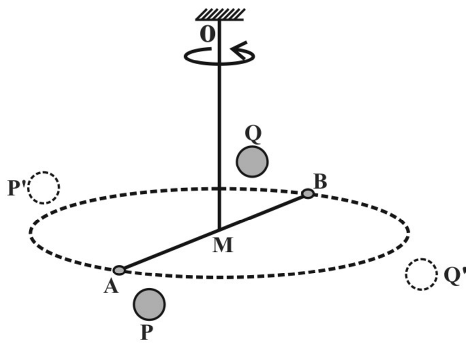
4 Universal Constant of Gavitation:

→ The value of the constant G appearing in the formula Gm_1m_2/r^2 showing Newton's universal law of gravitation, was first determined by Cavendish an English Scientist experimentally in 1798. The experimental arrangement is schematically shown in following figure.



Arrangement of Cavendish's experiment

→ From a rigid support a long rod is suspended using a thin metallic wire. Two small equal lead spheres A and B are attached at the ends of the rod. Two other equal large lead spheres are brought near the small spheres on opposite sides at equal distances.

→ The forces on the small spheres due to the large spheres are equal in magniude and opposite in directions. These forces produce torque. Hence the rod rotates about wire OM. Thus wire OM is twisted and the restoring torque (due to elasticity) is produced in the wire.

→ When the torque due to the gravitational forces equals the restoring torque, this system becomes steady (i.e. it comes in equilibrium).

→ In this condition the positions of large spheres P and Q (or P' and Q') are on lines perpendicular to AB.

> When position of large sphere is taken to P' and Q', the direction of torque will be reversed so the wire will be twisted.

→ Suppose, mass of each large sphere = M
mass of each small sphere = m

→ Distance between their centres in equilibrium condition = AP = BQ = r.
Angle of twist in the wire in equilibrium condition = θ
The restoring torque per unit twist = k

> tortional constant (N m / rad)

→ Length of rod, AB = l.

→ The gravitational force on the small sphere due to the large sphere $= \dfrac{GMm}{r^2}$ ——(1)

The total torque (moment of force couple) due to both such forces

> magnitude of one force x perpendicular distance.

$$\tau = \left[\frac{GMm}{r^2}\right](l) \qquad \text{——(2)}$$

and the restoring torque $\tau = k\theta$ ——(3)

In equilirium condition

$$\left[\frac{GMm}{r^2}\right](l) = k\theta \qquad \text{——(4)}$$

$$\therefore \boxed{G = \frac{k\,\theta\, r^2}{M\,m\,l}} \qquad \text{——(5)}$$

- Here the value of θ is obtained with the help of a small mirror attached to the wire, using lamp and scale method. These are not shown in figure.
- Moreover the value of k is obtained from some separate experiment of other kind in which known torque τ is applied and the twist in the wire θ is measured which gives $k = \dfrac{\tau}{\theta}$

→ Thus by measuring θ, G can be evaluted.

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5 Gravitation Acceleration And Variations In It :

5.1 (a) Acceleration due to gravity :

→ **"The acceleration produced in the body due to the gravitational force is called the gravitational acceleration or the acceleration due to gravity (g)."**

→ Considering Earth as a perfect sphere of uniform density we shall consider the acceleration due to Earth's gravity at different points.

→ We can imagine Earth to be made up of innumerable concentric(★) hollow spherical shells. Now a particle outside the Earth **is also outside all** these shells. Hence, to find the gravitational force on that particle, we can consider the mass of every shell as concentrated at the center of Earth. Thus to find the force on that particle due to entire Earth, we can consider the entire mass of the Earth to be concentrated at its centre.

→ Let the mass of Earth be M_e and radius be R_e. The gravitational force of earth on the particle of mass m, at distance r from the centre.

(here $r > R_e$); is $F = \frac{G M_e m}{r^2}$ ——— (1)

→ Hence, from Newton's second law of motion we can write, the acceleration due to gravity

Doesn't depend on the mass or volume of the object

$g = \frac{F}{m}$(✿) ∴ $g = \frac{G M_e}{r^2}$ ——— (2)

→ Now, for the particle on the surface of Earth , $r = R_e$

(★) Same centre

(✿) $F = ma \Rightarrow a = \frac{F}{m} \Rightarrow acceleration = \frac{\text{force}}{\text{mass}}$.

∴ Acceleration due to gravity for the particle on the Earth's surface is

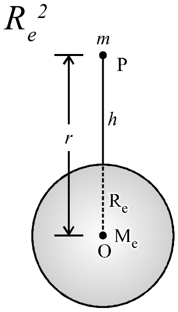
$$g_e = \frac{G M_e}{R_e^2} = \frac{4}{3} \pi G R_e \rho \quad \text{——— (3)}$$

- As we have considered the Earth as a perfect sphere the value of g_e at all points on the Earth's surface would be the same. (assume R_e is equal)
- In fact, Earth is not completely spherical but is slightly bulged out at the equator and flatterned at the poles. The radius of Earth at equator is nearly 21 km more than the radius at the poles. Hence the value of g_e at the poles is slightly more than that at the equator.
- But the variation in the value of g_e at different places on Earth's surface is extremely small and hence for practical purposes the value of g_e at every point on the Earth's surface is taken the same.
- The empirical value of g_e is found to be equal to 9.8 m/s^2.(★)

5.2 (b) Variation in gravitational accleration g with altitude :

5.2.1 The acceleration due to gravity at the Earth's surface is given by,

$$g_e = \frac{G M_e}{R_e^2} \quad \text{——— (1)}$$



Gravitational acceleration at height h from the Earth's surface

(★) You may calculate the value of g_e by taking $M_e = 6 \times 10^{24}$ kg and R_e = 6400 km in the above equation.

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→ The point P at height h from the Earth's surface is at distance $r = R_e + h$ from the centre of the Earth.

∴ The gravitational force of the Earth on a body of mass m at this point is

$$F(h) = \frac{G M_e m}{(R_e + h)^2} \quad \text{——— (2)}$$

∴ at P gravitational acceleration is

$$g(h) = \frac{F(h)}{m} = \frac{G M_e}{(R_e + h)^2} \quad \text{—— (3)}$$

(same as GM_e/r^2 where $r > R_e$ and $g \propto 1/r^2$)

$$\therefore \frac{g(h)}{g_e} = \frac{R_e^2}{(R_e + h)^2}$$

$$\therefore g(h) = \frac{R_e^2}{R_e^2\left[1 + \frac{h}{R_e}\right]^2} \quad \text{——— (4)}$$

$$\therefore g(h) = \frac{g_e}{\left[1 + \frac{h}{R_e}\right]^2} \quad \text{——— (5)}$$

- It is clear from this that $g(h) < g_e$

in denominator $\left[1 + \frac{h}{R_e}\right]^2$ so,

5.2.2 from equation (5),

$$g(h) = g_e\left[1 + \frac{h}{R_e}\right]^{-2}$$

$$= g_e\left[1 - \frac{2h}{R_e} + \frac{h}{R_e}\right.$$ terms with powers greater than 1.] .... (using binomial theorem).

→ If $h << R_e$, we can neglect the terms having powers greater than 1 of $\frac{h}{R_e}$. In such a condition

at height h decrease in g is $g_e - g(h) = 2\, g_e h / R_e$

$$g(h) = g_e\left[1 - \frac{2h}{R_e}\right] \quad \text{——— (6)}$$

- Equation (5) can be used for any height (h) but equation (6) can be used only when $h << R_e$.

5.2.3 We can take the value of g almost equal to g_e for small heights from the Earth's surface.

→ Let us understand this by an example : To find g for h = 10 km height from Earth's surface, we put R_e = 6400 km and $g_e = 9.8\ m/s^2$ in equation (6)

$$\therefore g\,(h = 10\text{ km}) = 9.8\left[1 - \frac{(2)(10)}{6400}\right]$$

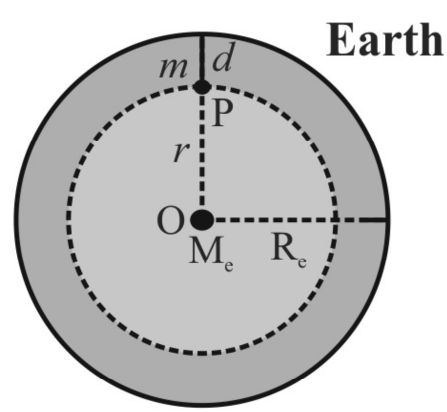
$$= 9.8 - 0.028$$

$$= 9.772$$

$$= 9.8\ m/s^2.$$

- Thus on the Earth's entire suface and for small heights from surface we can take $g = g_e = 9.8\ m/s^2$ for practical purposes.

5.3 Variation in the gravitational acceleration g with depth from the surface of the Earth :



Variation of g with depth from Earth's surface

→ Consider a particle of mass m at point P at a depth d from the suface of the Earth.

→ It is at distance $r = R_e - d$ from the centre of the earth.

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→ To find the Earth's gravitational force on this particle, we can imagine the Earth as made up of a small solid sphere of radius $r = R_e - d$ and a spherical shell of thickness d over it.

→ This paricle at point P is situated inside this hollow spherical shell.

→ Hence, the gravitational force on this particle due to the shell is zero.

→ Moreover, this particle is also on the **outer surface** of the small sphere (shaded) of radius r.

→ Hence the gravitational force on this particle can be obtained by considering the entire mass (M') of the **small sphere** at its centre O.

$$g(r) = \frac{4}{3} \pi G \rho r \quad \text{(1)}$$

Remember

→ From this equation, the gravitational acceleration at the surface of the Earth (putting $r = R_e$) is

$$g_e = \frac{4}{3} \pi G \rho R_e \quad \text{(2)}$$

→ From the equations (1) and (2)

$$\frac{g(r)}{g_e} = \frac{r}{R_e} \quad \text{(3)}$$

$$\therefore \quad g(r) = g_e\left(\frac{r}{R_e}\right) \quad \text{(4)}$$

$g \propto r$

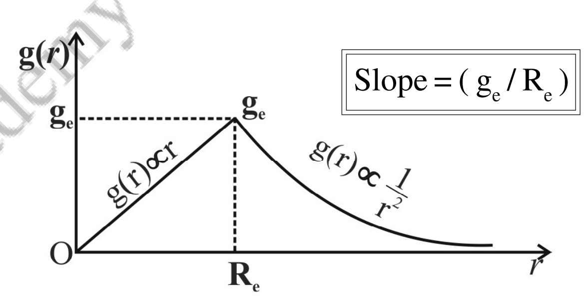
→ From equation (1) and (4) it is clear that g(r) is proportional to distance r from the centre of Earth upto the surface. Thus, the gravitational acceleration at a point inside the Earth is directly proportional to the distance of that point from the centre of the Earth.

→ Moreover, for region outside the Earth, $g(r) = GM_e / r^2$, shows that $g(r) \propto 1/r^2$.

→ Hence starting from the centre of the Earth, g(r) incresases in direct proportion as r increases. at centre g = 0 because r = 0

→ Then outside the surface g(r) decreases as inverse square of distance.

→ Such variation in g are shown in following figure.



variation in g with distance r from centre of earth

→ By substituting $r = R_e - d$ in equation (4) the gravitational acceleration is obtained in terms of depth d from the Earth's surface. We denote it as g(d).

$$\therefore \quad g(d) = \frac{g_e}{R_e} (R_e - d)$$

$$g(d) = g_e\left[1 - \frac{d}{R_e}\right] \quad \text{(5)}$$

Decrease in g at depth d is $g_e - g(d) = g_e d/R_e$.

- This shows that the gravitational acceleration at depth d has a smaller value than that at the Earth's surface.

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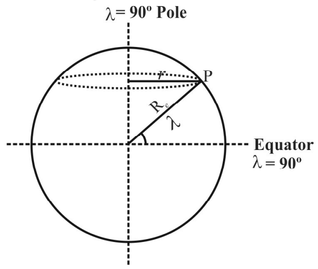
- Thus the acceleration due to Earth's gravity is maximum on its surface and from there on going above or below it decreases. It becomes zero(★) at the Earth's centre. This is a notable fact.

➥ **Variation in g with latitude :**



$r = R_e \cos\lambda$

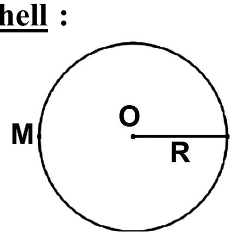
→ The angle made by the line joining a given place on the Earth's surface to the centre of the Earth with the equitorial line is called the latitude (λ) of that place.

→ Effective gravitational acceleration at a place where latitude is λ,
put $r = R_e \cos\lambda$ in $g' = g_e - r\,\omega^2 \cos\lambda$.
$\therefore$ $\boxed{g' = g_e - R_e\,\omega^2 \cos^2\lambda.}$
at equator (minimum)
$g' = g_e - R_e\,\omega^2$ ($\because \lambda = 0$) and
at poles (maximum)
$g' = g_e$ ($\because \lambda = 90^0$)

→ If the rotation of the Earth stops, $\omega = 0$ and $g' = g_e$
mean gravitational acceleration increases by $R_e\omega^2\cos^2\lambda$.

→ If we are going towards poles from equator λ increases so cosλ decreases and thats why effective gravitational acceleration increases.

Shell :



$g = 0$ ($r < R$)
$g = (GM / R^2)$ ($r = R$)
$g = (GM / r^2)$ ($r > R$)

(★) decrease in g with hight is more speedy than decrease in g with depth.

Notes

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Solved Problems

(34) At what height the weight of the object is 1/16 times compare to the weight of that object at the surface of the earth (radius R) ? [NEET : 2012]

(A) 5 R (B) 15 R (C) 3 R (D) 4 R

Solution : **[Ans. : C]**

→ According to question,

$$\frac{GMm}{(R+h)^2} = \frac{1}{16}\frac{GMm}{R^2}$$

where m = mass of body.

→ $$\frac{1}{(R+h)^2} = \frac{1}{16\ R^2}$$

→ $$\frac{R}{R+h} = \frac{1}{4}$$

→ $R + 4 = 4R$

→ $h = 3R$

(35) If the Earth were a sphere made completely of gold (!), what would have been the magnitude of gravitational acceleration on its surface ?
The radius of the Earth = 6400 km, density of gold = 19.3 × 10³ kg/m³.
G = 6.67 × 10⁻¹¹ Nm²/kg².

Solution :

→ $R_e = 6400\text{ km} = 64 \times 10^5\text{ m}$

$\varrho = 19.3 \times 10^3\ \text{kg/m}^3$

$g_e = ?$

→ $$g_e = \frac{GM_e}{R_e^2}$$

$$g_e = \frac{G(\frac{4}{3}\pi R_e^3 \varrho)}{R_e^2} \quad (\because \text{mass} = \text{volume} \times \text{density})$$

$$g_e = \frac{4}{3} G \pi R_e \varrho$$

$$= \frac{4}{3}(6.67\times 10^{-11})(3.14)(64\times 10^5)(19.3\times 10^3)$$

$$\boxed{g_e = 34.49\ \text{m/s}^2}$$

(36) Assume that a new planet were made form a material having density same as that of Earth. But the volume of the planet would be three times that of Earth then how much the gravitational acceleration of the new planet ?

Solution :

→ $$\frac{g_p}{g_e} = \frac{m'}{m_e}\frac{R_e^2}{R_e'^2}$$

→ $V' = 3V$ (given)

$$\therefore \frac{4}{3}\pi R_e'^3 = 3\left(\frac{4}{3}\pi R_e^3\right)$$

$$\therefore R_e'^3 = 3R_e^3$$

$$\therefore \frac{R_e}{R'_e} = \frac{1}{3^{1/3}}$$

→ $$\frac{g_p}{g_e} = \frac{m'}{m_e} \times \frac{R_e^2}{R_e'^2}$$

$$= \frac{3V\varrho}{V\varrho}\left(\frac{R_e^2}{R_e'^2}\right)$$

$$= \frac{3R_e^2}{R_e'^2} = 3\left(\frac{R_e}{R'_e}\right)^3 = 3\left(\frac{1}{3^{1/3}}\right)^2$$

$$= 3^{1/3}$$

$$= (3^{1/3})\, g_e = 1.443 \times 9.8$$

$$g_p = 14.14\ \text{ms}^{-2}$$

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(37) A mango of 300 g falls down from the mango tree. Then find the gravitational accelration of the mango toward Earth. Also find the gravitational acceleration of the Earth toward mango.

Take $M_e = 6 \times 10^{24}$ kg,

R_e = 6400 km and

$G = 6.67 \times 10^{-11}$ Nm²/kg².

Solution :

$\rightarrow$ m = 0.3 kg

$M_e = 6 \times 10^{24}$ kg

R_e = 6400 km

$G = 6.67 \times 10^{-11} \dfrac{Nm^2}{Kg^2}$

$\rightarrow \quad F = \dfrac{GM_e m}{R_e^2}$

$mg = \dfrac{GM_e m}{R_e^2}$

$g = \dfrac{GM_e}{R_e^2}$

$g = 9.8 \text{ m/s}^2$

$\rightarrow \quad M_e g_m = \dfrac{GM_e m}{R_e^2}$

$g_m = \dfrac{6.67 \times 10^{-11} \times 0.3}{(64)^2 \times 10^{10}}$

$g_m = 4.9 \times 10^{-25} \text{ m/s}^2$

(38) Radii of two planets are r_1 and r_2 respectively and their densities are ρ_1 and ρ_2 respectively. The gravitational accelerations on their surfaces are g_1 and g_2 resectively. $\therefore$ (g_1/g_2) = ...

(A) $\dfrac{r_1 \rho_1}{r_2 \rho_2}$ **(B)** $\dfrac{r_2 \rho_2}{r_1 \rho_1}$

(C) $\dfrac{r_1}{r_2} \cdot \dfrac{\rho_2}{\rho_1}$ **(D)** $\dfrac{r_2}{r_1} \cdot \dfrac{\rho_1}{\rho_2}$

Solution : [Ans. : A]

$\Rightarrow \quad g = \dfrac{Gm_1}{r^2} = \dfrac{G}{r^2} \times \dfrac{4}{3} \pi r^3 \rho$

(where $m = \dfrac{4}{3} \pi r^3 \rho$)

$\therefore \quad g = (4/3)\, \pi\, G\, r\rho$

on first planet $g_1 = (4/3)\, \pi\, G\, r_1 \rho_1$

on second planet $g_2 = (4/3)\, \pi\, G\, r_2 \rho_2$

$\therefore \quad \dfrac{g_1}{g_2} = \dfrac{r_1 \rho_1}{r_2 \rho_2}$

(39) The density of newly discovered planet is twice that of earth. The acceleration due to gravity at the surface of the planet is equal to that at the surface of the earth. If the radius of the earth is R, the radius of the planet would be :

[NEET : 2004]

(A) 2 R (B) 4 R (C) $\frac{1}{4}$ R (D) $\frac{1}{2}$R

Solution : [Ans. : D]

$\rightarrow \quad g = \dfrac{F}{m}$

$\therefore \quad F = \dfrac{GMm}{R^2}$

$\left[M_p = \dfrac{4}{3} \pi R_p^3 \rho_p ,\ M_e = \dfrac{4}{3} \pi R_e^3 \rho_e \right]$

$\therefore \quad \rho_p = 2\rho_e$

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$g_p = g_e$

$\therefore \quad \frac{GM_p}{R_p^2} = \frac{GM_e}{R_e^2}$

$\text{Density} = \frac{\text{mass}}{\text{volume}}$

$$\frac{G(\frac{4}{3}\pi R_p^3 \rho_p)}{R_e^2} = \frac{4(\frac{4}{3}\pi R_e^3 \rho_e)}{R_e^2}$$

$R_p \rho_p = R_e \rho_e$

$R_p \times 2\rho_e = R_e \rho_e$

$\therefore \quad R_p = \frac{R_e}{2}$

$R_p = \frac{R}{2}$

(40) If the radius of the Earth suddenly decreases to 60% of the present value (with mass of the Earth remaining the same) what would be the percentage change in the magnitude of the gravitational accleration g_e, on the surface of the Earth ?

Solution :

→ Original value of gravitational acceleration $g_e = \frac{GM_e}{R_e^2}$

→ New radius of Earth $R_e' = \frac{60}{100} R_e$

$= 0.6\ R_e$

If radius decreases by 60 % then remaining will be 40 %, so take $R_e' = 0.4\ R_e$

$\therefore$ New value of gravitational acceleration

$g_e' = \frac{GM_e}{R_e'^2}$

$g_e' = \frac{GM_e}{(0.6\ R_e)^2} = \frac{g_e}{0.36} = \frac{25}{9} g_e$

(♠) For CTP 76, We have, (dg/g) x 100 % = – 2[(dr/r) x 100 %] which is used up to 9% variation

$\therefore$ Increase in the gravitational acceleration

$\Delta g = g_e' - g_e = \frac{25}{9} g_e - g_e = \frac{16}{9} g_e$

$= 1.778\ g_e$

$\therefore$ Percentage increase in the magnitude of gravitational acceleration

$= \frac{\text{increase}}{\text{original value}} \times 100$

$= 1.778 \times 100 = 177.8\ \%$

(41) If the mass and the radius of the Earth both decreases by 1 %, what will be the percentage change in the gravitational accleration at the surface ?

Solution :

→ The original value of acceleration

$g_e = \frac{GM_e}{R_e^2}$

→ If $M_e' = 0.99\ M_e$ and $R_e' = 0.99\ R_e$, then new value of gravitational acceleration

$g_e' = \frac{GM_e'}{R_e'^2} = \frac{G \times 0.99 M_e}{(0.99 R_e)^2}$

$= 1.01 \left(\frac{GM_e}{R_e^2}\right)$

$g_e' = 1.01\ g_e$

$\therefore$ Change in the gravitational acceleration

$= g_e' - g_e = 1.01\ g_e - g_e = 0.01\ g_e$

$\therefore$ Percentage change in the gravitational acceleration

$= \frac{\text{change}}{\text{original value}} \times 100 = \frac{0.01\ g_e}{g_e} \times 100$

$= 1\ \%$

→ Thus the magnitude of g_e increases by 1 %.(♠)

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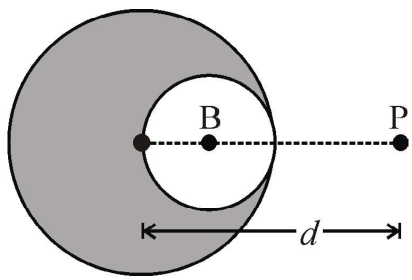
(42) As shown in the figure, from an iron sphere of mass M and radius R; a small sphere of diameter R is cut. Find the force of gravitational on the body of mass m due to the remaining sphere at a point at distance d from the centre of the original sphere on the line joining the centres of the two spheres.



Solution : **[Ans. : B]**

→ For big sphere : radius R, mass M , volume V.

→ Volume of small sphere of radius (R/2)

$$V' = \frac{4}{3}\pi\left(\frac{R}{2}\right) = \frac{4}{3}\ \pi R^3 \times \frac{1}{8} = \frac{V}{8}$$

Fmass of small sphere, $M' = \frac{M}{8}$

→ REsultant force at point P,

$$F = \begin{pmatrix}\text{gravitational}\\\text{force due to big}\\\text{sphere}\end{pmatrix} - \begin{pmatrix}\text{gravitational}\\\text{force due to}\\\text{small sphere}\end{pmatrix}$$

$$F = \frac{GMm}{d^2} - \frac{GM'm}{\left(d-\frac{R}{2}\right)^2}$$

$$F = \frac{GMm}{d^2} - \frac{\frac{GMm}{8}}{d^2\left(1-\frac{R}{2d}\right)^2}$$

$$= \frac{GMm}{d^2}\left[1-\frac{1}{\frac{8\,(2d-R)^2}{4d^2}}\right]$$

$$\therefore \quad \therefore F = \frac{GMm}{d^2}\left[1-\frac{d^2}{2(2d-R)^2}\right]$$

(43) Read the following paragraph and answer the questions that follow :

The reason for the tides in the ocean is the gravitation. In this phenomenon gravitation of the sun and the moon both play part. Actually the gravitational force by the sun on the Earth is nearly 175 times that exerted by the moon on the Earth. However in the phenomenon of tides the contribution by the moon is more than that by the sun - it is nearly 2.17 times that by the sun. What could be the reason for this? The reason for this is that the calculations reveal that the tide generating force (tidal force) depends on the rate of the change of gravitational force with distance and not on the magnitude of the gravitational force itself. Hence in spite of

$F_{\text{by sun}} > F_{\text{by moon}}$ **since** $d/dr\,(F_{\text{by moon}}) > d/dr\,(F_{\text{by sun}})$

the contribution by the moon is more in the phenomenon of tides.

$$F = \frac{GMm}{r^2} \text{ gives } \frac{d}{dr}(F) = \frac{-2GMm}{r^3}$$

From this the tidal force is found to be proportional to M/r^3 when M = mass of sun or moon and r = their distance from Earth.

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Questions :

(i) In the tides in the ocean; out of the sun and the moon, which has greater contribution ?

(ii) If we find M/r^3 for the sun and for the moon; which would be greater ?

(iii) What is the relative strength of the gravitational forces on the Earth by the sun and by the moon ?

Solution :

(i) In the phenomenon of tides the contribution by the moon is more then that by the sun is nearly 2.17 times that by the sun because the field force is in proportion of $\frac{M}{r^3}$.

(ii) The magnitude of $\frac{M}{r^3}$ is more than that of the moon.

(iii) $F_{(by\ sun)} = 175\ F_{(by\ moon)}$

(44) Answer the following :

(a) You can shield a charge from electrical forces by putting it inside a hollow conductor. Can you shield a body from the gravitational influence of near by matter by putting it inside a hollow sphere or by some other means?

(b) An astronaut inside a small space ship orbiting around the earth cannot detect gravity. If the space station orbiting around the earth has a large size, can he hope to detect gravity ?

(c) If you compare the gravitational force on the earth due to the sun to that due to the moon, you would find that the Sun's pull is greater than the moon's pull. (you can check this yourself using the data available in the succeeding exercises). However, the tidal effect of the moon's pull is greater than the tidal effect of sun. Why ?

Solution :

(a) No, the gravitaional force on a body due to near by matter is independent of the presence of other matter. It means gravitational screenings are not possible.

(b) Yes, if the size of the spaceship orbiting around the earth is large enough an astronaut inside the space ship can detect the variation in g.

(c) The tidal effect depends inversly upon the cube of the distance, where as the gravitaional force varies inversly with the square of the distance. As the moon is closer to earth than the sun, so its tidal effect is greater than that of the sun.

Ratio :

$$\frac{T_m}{T_s} = \left(\frac{d_s}{d_m}\right)^3$$

where T shows the tidal effect

$$= \left(\frac{1.5 \times 10^{11}}{3.8 \times 10^{8}}\right)^3$$

$$= 61.5 \times 10^6$$

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(45) Suppose the gravitational force varies inversly as the n^{th} power of distance. Then the time period of a planet in circular orbit of radius 'R' around the sun will be proportional to ....

[JEE : 2004]

(A) R^n **(B) $R^{(n-1/2)}$**

(C) $R^{(n+1/2)}$ **(D) $R^{(n-2/2)}$**

Solution : **[Ans. : C]**

→ Centripital force (F_C)

= gravitational force (F_g)

$$\frac{mv^2}{R} = G\frac{Mm}{R^n}$$

$$\therefore v^2 = \frac{GM}{R^{n-1}}$$

$$\therefore \frac{4\pi^2 R^2}{T^2} = \frac{GM}{R^{n-1}} \qquad \left(\therefore v = \frac{2\pi R}{T}\right)$$

$$\therefore T^2 = \frac{4\pi^2}{GM} R^{n+1}$$

$$T = \left(\frac{4\pi^2}{GM}\right)^{1/2}\left(R^{\frac{n+1}{2}}\right)$$

$$T \propto R^{\frac{n+1}{2}}$$

(46) Two astronauts are floating in gravitational free space after having lost contact with their spaceship. The two will : **[NEET : 2017]**

(A) keep floating at the same distance between them

(B) move away from each other

(C) move towards each other

(D) will become stationary

Solution : **[Ans. : C]**

(47) Two masses m_1 and m_2 ($m_1 < m_2$) are released from rest from a finite distance. They start under their mutual gravitational attraction :

[AIIMS : 2012]

(A) acceleration of m_1 is more than that of m_2

(B) acceleration of m_2 is more than that of m_1

(C) centre of mass of system will remain at rest in all the reference fram

(D) total energy of system does not remain constant

Solution : **[Ans. : A]**

(48) If the mass of the Sun were ten times smaller and the universal gravitational constant were ten times larger in magnitude, which of the following is not correct ? **[NEET : 2018]**

(A) g on the Earth will not change.

(B) Raindrops will fall faster.

(C) Tine period of a simple pendulum on the Earth would decrease.

(D) Walking on the ground would become more difficult.

Solution : **[Ans. : A]**

(49) The height at which the acceleration due to gravity becomes (g / 9) (whsere g = the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth, is : [JEE : 2009]

(A) R / $\sqrt{2}$ **(B) R/2**

(C) $\sqrt{2}\ R$ **(D) 2R**

Solution : **[Ans. : D]**

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→ From, $g(h) = \dfrac{g_e}{\left[1 + \dfrac{h}{R_e}\right]^2}$

$$g(h) = \frac{g_e}{9} \quad , \; h = 2\,R_e$$

(50) The change in the value of 'g' at a height 'h' above the surface of the earth is the same as at a depth 'd' below the surface of earth. When both 'd' and 'h' are much smaller than the radius of earth, then which one of the following is correct ? [AIEEE : 2005]

(A) $d = 3\,h / 2$ **(B) $d = h / 2$**

(C) $d = h$ **(D) $d = 2\,h$**

Solution : [Ans. : D]

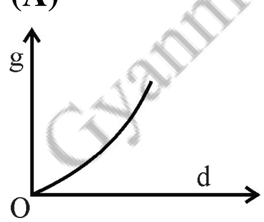
→ g (d) = g (h)

$$g_e\left(1-\frac{d}{R_e}\right) = g_e\left(1-\frac{2h}{R_e}\right)$$

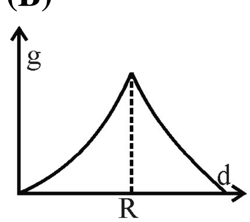
$$\left(1-\frac{d}{R_e}\right) = \left(1-\frac{2h}{R_e}\right) \Rightarrow d = 2h$$

(51) The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius) : [AIEEE : 2017]

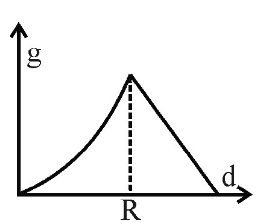
(A)



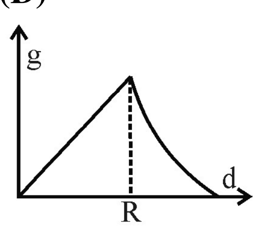
(B)



(C)



(D)



Solution : [Ans. : D]

→ Inside the earth $g \propto d$.

→ On the surface of earth $g = \dfrac{GM}{R^2}$

→ For outside earth $g \propto \dfrac{1}{d^2}$

(52) If we take the gravitational acceleration at the Earth's surface as 10 m/s^2 and radius of the Earth as 6400 km, the decrease in the value of the gravitational acceleration g at a depth of 64 km from its surface would be ....

(A) 0.1 m/s^2 **(B) 0.2 m/s^2**

(C) 0.05 m/s^2 **(D) 0.3 m/s^2**

Solution : [Ans. : A]

→ Gravitational acceleration from the surface of the Earth at height d,

$$g = g_e\left(1 - \frac{d}{R_e}\right)$$

$$= g_e - \frac{d}{R_e}\,g_e$$

$$\therefore \quad g - g_e = -\frac{dg_e}{R_e}$$

$$= -\frac{64 \times 10}{6400}$$

$\therefore \quad g - g_e = -0.1\ m/s^2$

∴ Negative sign shows decrease in gravitational acceleration

OR

$$\boxed{\text{decrease} = g_e d/R_e}$$

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(53) The acceleration due to gravity on the planet A is 9 times the acceleration due to gravity on planet B. A man jumps to a height of 2m on the surface of A. What is the height of jump by the same person on the planet B ?

[NEET : 2003]

(A) 6 *m* **(B) 2 / 3 *m***
(C) 2 / 9 *m* **(D) 18 *m***

Solution : **[Ans. : D]**

→ $g_A = 9\, g_B$ (given)

$v^2 = 2gh$

For planet A, $h_A = \frac{v^2}{2g_A}$ ... ...(1)

For planet B, $h_B = \frac{v^2}{2g_B}$ ... ...(2)

dividing eqn (2) by eq^n (1)

→ $\frac{h_A}{h_B} = \frac{g_B}{g_A}$

→ $\frac{h_A}{h_B} = \frac{g_B}{9\, g_B} = 1/9$

→ $h_B = 9\, h_A$

$= 9 \times 2$

$h_B = 18$ m

(54) A man travels a distance of 60 m initially in the free fall from near the Earth's surface. What distance would be travel initially in the free fall near the moon's surface in the same time ?

($g_m = g_e / 6$)

(A) 60 *m* **(B) 360 *m***
(C) 100 *m* **(D) 10 *m***

Solution : **[Ans. : D]**

→ Distance travelled by an object in time t, falling freely near the earth's surface

$d_e = \frac{1}{2} g_e t^2$

Distance travelled by an object in time t, falling freely near the moon surface

→ $d_m = \frac{1}{2} g_m t^2$

→ $\frac{d_m}{d_e} = \frac{g_m}{g_e}$

→ $d_m = d_e \times \frac{g_m}{g_e}$

$= 60 \times \frac{g_e}{6g_e}$

$d_m = 10$ m

(55) Imagine earth to be a solid sphere of mass M and radius R. If the value of acceleration due to gravity at a depth 'd' below earth's surface is same as its value at a height 'h' above its surface and equal to (g / 4) (where g is the value of acceleration due to gravity on the surface of earth), the ratio of (h / d) will be :

[NEET : 2017]

(A) 1 **(B) 4 / 3** **(C) 3 / 2** **(D) 2 / 3**

Solution : **[Ans. : B]**

→ $g(h) = \frac{g}{\left(1+\frac{h}{R_e}\right)^2}$

$\frac{g}{4} = \frac{g}{\left(1+\frac{h}{R_e}\right)^2}$

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$$\therefore \frac{h}{R_e} = 1 \quad \dots\dots\dots(1)$$

$$\rightarrow \quad g(d) = g\left(1 - \frac{d}{R_e}\right)$$

$$\frac{g}{4} = g\left(1 - \frac{d}{R_e}\right)$$

$$\therefore \frac{d}{R_e} = 1 - \frac{1}{4}$$

$$\therefore \frac{d}{R_e} = \frac{3}{4} \quad \dots\dots\dots(2)$$

From (1) & (2),

$$\frac{h}{d} = \frac{4}{3}$$

(56) Person 'A' standing on the surface of the Earth and persons B and C are standing on at the top of the Aifil tower and at the depth as the height of aifil tower resectively. If accelerations due to gravity acting upon them are g_A, g_B and g_C respectively then

(A) $g_A > g_B > g_C$

(B) $g_B > g_A > g_C$

(C) $g_A > g_C > g_B$

(D) $g_C \geq g_B > g_A$

Solution : **[Ans. : C]**

(57) Calculate the fractional decrement in the weight of the object when it is taken to the 1600 km downwards in the earth. Radius of the earth is 6400 km.

(A) 25 % **(B) 30 %**

(C) 40 % **(D) 15 %**

Solution : **[Ans. : A]**

→ Change in weigth of object,

$$m(g - g_e) = -\frac{mdg_e}{R_e}$$

$$m(g - g_e) = -m\left(\frac{1600 \times 10^3 \times 10}{6400}\right)$$

$$= \frac{10^4}{4}$$

$$= 25\%$$

58) Find the period of rotation of the Earth about its own axis in terms of R_e and g for which the effective acceleration due to gravity becomes zero at the equator?

Also can be asked that if effective weight is zero ?

Solution :

→ At the equator the latitude $\lambda = 0°$.

- The effective acceleration due to gravity at a place having latitude λ on the Earth's surface is given by

  $g' = g - R_e\omega^2\cos^2\lambda$

- R_e = radius of the Earth
- g = gravitational acceleration at the Earth's surface without considering rotation.
- ω = angular speed of Earth's rotation $= \frac{2\pi}{T}$.

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→ We want to find time-period T for g'= 0 at the equator.

$\therefore\ 0 = g - R_e\omega^2\cos^2(0°)$

$\therefore\ g = R_e\omega^2 \quad (\cos 0° = 1)$

$= R_e\left[\frac{4\pi^2}{T^2}\right]$

$\therefore\ T^2 = 4\pi^2\frac{R_e}{g}$

$\therefore\ \boxed{T = 2\pi\sqrt{\frac{R_e}{g}}}$ (★) $\quad \boxed{\omega = \sqrt{\frac{g}{R_e}}}$

(59) What would be the fictitious (pseudo) acceleration of the body lying on the equator of Earth in the radial direction away from the Earth's centre due to its rotation?

(A) ωR_e **(B) $\omega^2 R_e$**

(C) ωR_e^2 **(D) $\omega^2 R_e^2$**

Solution : **[Ans. : B]**

→ Where, ω = angular speed of the earth,

R_e = radius of the earth

centripetal acceleration $a_c = v^2 / R_e$

but put $v = R_e\omega$

$a_c = R_e^2\omega^2 / R_e = R_e\omega^2$

(★) Obtained T for λ = 45⁰ lattitude
Answer will be,

$T = 2\pi\sqrt{\frac{R_e}{g}}$

Current Topic Practice

(60) What will be the formula of the mass in terms of g, R and G ? (R = radius of earth)

[NEET : 1996]

(A) $\frac{g^2R}{G}$ (B) $\frac{GR^2}{g}$

(C) $\frac{GR}{g}$ (D) $\frac{gR^2}{G}$

(61) If the radius of the earth were to shrink by one percent, its mass remaining the same, the acceleration due to gravity on the earth's surface would........

[IIT : 1981]

(A) decrease (B) remain unchanged

(C) increase (D) be zero

(62) The mass and diameter of a spherical planet is M_P and D_P respectively the acceleration due to gravity experienced by the objects of mass m is ....

[NEET : 2012]

(A) $\frac{4GM_P}{D_P^2}$ (B) $\frac{GM_P m}{D_P^2}$

(C) $\frac{GM_P}{D_P^2}$ (D) $\frac{4GM_P m}{D_P^2}$

(63) If the Earth were a sphere made completely of iron (!), what would have been the magnitude of gravitational acceleration on its surface ?

The radius of the Earth = 6.37×10^6 m

density of iron = 7.86×10^3 kg/m³.

$G = 6.67 \times 10^{-11}$ Nm²/kg².

Answer : (60) D (61) C (62) A

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(64) The gravitational acceleration on the surface of moon is 1.67 ms^{-2}. If the radius of moon is 1.74×10^6 then find the mass of the moon.

Take $G = 6.67 \times 10^{-11}\ Nm^2/kg^2$.

(65) Average density of the earth ....

[AIEEE : 2005]

(A) is a complex function of g

(B) does not depend on g

(C) is inversely proportional to g

(D) directly proportional to g

$$g = \frac{4}{3}\pi G \rho r \quad \therefore \ \rho \propto g$$

(66) Imagine a new planet having the same density as that of earth but it is 3 times bigger than the earth in size. If the acceleration due to gravity on the surface of earth is g and that on the surface of the new planet is g', then [NEET : 2005]

(A) g' = 3g (B) g' = g / 9

(C) g' = 9g (D) g' = 27g

$$g = \frac{GM}{r^2} = \frac{G}{r^2}\rho\,(\text{volume}) \quad \therefore g \propto r$$

(67) If radii of two planets of equal density are equal and the accelerations due to gravity on its surfaces are g_1 and g_2 then which of the following is true ?

(A) $g_1 > g_2$ (B) $g_1 < g_2$

(C) $g_1 = g_2$ (D) All of the above

(68) Which of the following relation is correct for the acceleration due to gravity on the surface of the planet and its denstity.

(A) $g \propto (1/\rho)$ (B) $g \propto \rho^2$

(C) $g \propto \rho$ (D) $g \propto \sqrt{\rho}$

(69) If the densities of two equal radii planets are ρ_1 and ρ_2 than

(A) $\frac{g_1}{g_2} = \frac{\rho_1^2}{\rho_2}$ (B) $\frac{g_2}{g_1} = \frac{\rho_1}{\rho_2}$

(C) $\frac{g_1}{g_2} = \frac{\rho_1^2}{\rho_2}$ (D) $\frac{g_2}{g_1} = \frac{\rho_1^2}{\rho_2^2}$

(70) If the radii of two planets are r_1 and r_2 and their masses are m_1 and m_2 and their gravitational accelerations are g_1 and g_2 than m_1 / m_2 =

(A) g_1r_2 / g_2r_1 (B) $g_1r_1^2 / g_2r_2^2$

(C) g_2r_1 / g_1r_2 (D) $g_2r_2^2 / g_1r_1^2$

(71) Two planets having radii r_1 and r_2 respectively are made up of same material, then the ratio of the acceleration due to gravity on the surface of the planets (g_1 / g_2) is ....

(A) r_1 / r_2 (B) $2r_1 / r_2$

(C) r_2 / r_1 (D) $2r_2 / r_1$

(72) If the mass and the radius of the Earth both decreases by 2 % what will be the percentage change in the gravitational acceleration at the surface?

(73) If the mass of the Earth remaining the same and the radius decreases by 2%, what will be the percentage change in the gravitational acceleration at the surface?

(74) If the mass and the radius of the Earth both increases by 2 % what will be the percentage change in the gravitational acceleration at the surface ?

Answer : (65) D (66) A (67) C (68) C (69) C (70) B (71) A

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(75) If the mass of the Earth becomes 80 times that of any one planet and the diameter of that planet becomes 1/4 of the Earth then how much the gravitational acceleration on that planet?

(76) If the raidus of the Earth decreases by 0.5% remaining its mass constant. What would be the change in the value of g on its surface?

(A) increases by 1 %

(B) decreases by1 %

(C) increases by 0.5%

(D) decreases by0.5 %

(77) If the gravitational acceleration at the Earth's surface is 9.81 m/s^2, what is its value at a height equal to the diameter of the Earth from its surface?

(A) 4.905 m/s^2 (B) 2.452 m/s^2

(C) 3.27 m/s^2 (D) 1.09 m/s^2

(78) If the gravitational acceleration at the Earth's surface is 9.81 m/s^2, what is its value at a height equal to the radius of the Earth from its surface?

(A) 1.45 m/s^2 (B) 2.45 m/s^2

(C) 4 m/s^2 (D) 4.25 m/s^2

(79) If the value of gravitational acceleration at the Earth's surface is 9.81 m/s^2, then at what times of distance than the radius from the surface of the earth the gravitational acceleration will be 1.09 m/s^2 ?

(A) 2 (B) 3 (C) 4 (D) 6

(80) A planet which has mass and radius are half of the Earth, what wll be the acceleration due to graivity for that planet? Take g = 9.8 m/s^2

(A) 9.8 m/s^2 (B) 4.9 m/s^2

(C) 19.6 m/s^2 (D) 39.2 m/s^2

(81) What would be the value of acceleration due to gravity if we halved the radius of the earth keeping mass constant ?

(A) 2g (B) 3g

(C) 4g (D) 8g

(82) The mass of the earth is halved keeping radius constant then what would be the magnitude of the acceleration due to gravity ?

(A) 2g (B) g / 2

(C) g / 4 (D) g / 3

(83) The density of a planet having the radius same as of the earth is twice then the density of earth, then value of g on that planet will be ....

(A) $2g_e$ (B) $3g_e$

(C) g_e (D) $4g_e$

(84) The mass of the moon is 1/81 times than earth and the dimeter of the moon is (1 / 3.7) times than the earth. If the value of g on the surface of earth is 9.8 m/s^2 . then the value of g on the surface of the moon will be ....

(A) 1.83 (B) 1.63

(C) 1.43 (D) 1.23

(85) If a mass of a body is M on the earth surface, the mass of the same body on moon surace will be : [AIIMS : 1997]

(A) M (B) M / 6

(C) zero (D) none of these

Answer : (76) A (77) D (78) B (79) A (80) C (81) C (82) B (83) A (84) B (85) A

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(86) The acceleration due to gravity at a height 1 km above the earth is the same as at a depth d below the surface of earth. Then: [NEET - 2017]

(A) $d = \frac{1}{2}$ km
(B) d = 1 km
(C) d = (3 / 2) km
(D) d = 2 km

(87) The value of acceleration due to gravity, at earth surface is g. Its value at the centre of the earth, which we assume as a sphere of radius R and of uniform mass density, will be : [AIIMS : 1997]

(A) 10 R m / s^2 (B) zero
(C) 5 R m / s (D) 20 R m / s^2

(88) Taking the radius of the Earth as R; at what height above its surface the value of g will be half of its value on the surface ?

(A) $2\sqrt{2}$ R (B) $(\sqrt{2} - 1)$ R
(C) $(2 - \sqrt{2})$ R (D) R

(89) Weight of man at the Earth's surface is 500N. At what height from the Earth's surface would it be 250 N ?
(Radius of the Earth = 6400 km)

(A) 2525 *km* (B) 2650 *km*
(C) 3200 *km* (D) 6400 *km*

(90) Weight of an object on surface of eath is 72 N. At the height of half of the radius from surface, its weight will be ..... [NEET : 2000]

(A) 72 N (B) 28 N
(C) 10 N (D) 32 N

(91) If magnitude of gravitational acceleration at height d from the surface of the earth is 6.2 m/s^2, d = ... ? Take g = 9.8 m/s^2

(A) 2351 km (B) 4032 km
(C) 6100 km (D) 5800 km

(92) Considering the earth as a perfect sphere, calculate the fractional change in the weight of the object when it is taken to the height of 64 km from the surface of the earth. Take the radius of the earth = 6400 km.

(A) 2 % (B) 3 %
(C) 4 % (D) 1 %

(93) The angular speed of earth in rad/s, so that bodies on equator may appear weightless is : [Use g = 10 m/s^2 and the radius of earth = 6.4×10^3 km] [AIIMS : 2011]

(A) 1.25×10^{-3} (B) 1.56×10^{-3}
(C) 1.25×10^{-1} (D) 1.56

(94) What should be the angular velocity of Earth's rotation about its own axis so that the body at 45^0 lattitude at the Earth's surface becomes weightless. (g = gravitational acceleration at the Earth's surface, R = radius of Earth)

(A) $\sqrt{2g / R}$ (B) $\sqrt{g / R}$
(C) $\sqrt{3g / R}$ (D) $\sqrt{4g / R}$

(95) What would be the fictitious (pseudo) force on the body lying on the equator of Earth in the radial direction away from the Earth's centre due to its rotation?

(A) $M_e\omega R_e$ (B) $M_e\omega^2 R_e$
(C) $M_e\omega R_e^2$ (D) $M_e\omega^2 R_e^2$

Answer : (86) D (87) B (88) B (89) B (90) D (91) A (92) A (93) A (94) A (95) B

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(96) What would be the fictitious (pseudo) acceleration of the body lying on the pole. of Earth in the radial direction away from the Earth's centre due to its rotation?

(A) ωR_e (B) $\omega^2 R_e$

(C) 0 (D) $\omega^2 R_e^2$

(97) If the Earth suddenly stops rotating about its own axis then what change in the value of gravitational acceleration at the equator will be found from that obtained during its rotation ?

(A) Increases by ωR_e^2

(B) Increases by $\omega^2 R_e$

(C) Decreases by $\omega^2 R_e$

(D) Decrease by ωR_e^2

(98) What will be the effect on the weight of a body placed on the surface of earth, if earth suddenly stops rotating ? [AIIMS : 2014]

(A) No effect

(B) weight will increase

(C) weight will decrease

(D) weight will become zero

NOTE

Answer : (96) C (97) B (98) B

NOTE

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Hints & Solution

(60)

→ According to Newton's law of gravitation

$$F = \frac{GMm}{R^2}$$

and F = mg

$$\therefore\ mg = \frac{GMm}{R^2}$$

$$\therefore\ g = \frac{GM}{R^2}$$

$$\therefore\ M = \frac{gR^2}{G}$$

(62)

→ Gravitational force between particle of mass m and planet

$$F = \frac{GM_pm}{(D_p/2)^2}$$

$$= \frac{4\ GM_pm}{D_p^2}$$

→ Acceleration due to gravity,

$$a = \frac{F}{m}$$

$$= \frac{4GM_p}{D_p^2}$$

(63)

$$g_e = \left(G\frac{4}{3}\pi R_e\right)\rho$$

$$= 6.67 \times 10^{-11} \times \frac{4}{3} \times 3.14 \times 6.37 \times 10^6 \times 7.86 \times 10^3$$

$$g_e = 13.98\ m/s^2$$

(64)

$$g_m = \frac{GM_m}{R_m^2}$$

$$M_m = \frac{g_m R_m^2}{G}$$

$$= \frac{1.67 \times (1.74 \times 10^6)^2}{6.67 \times 10^{-11}}$$

$$M_m = 7.5 \times 10^{22}\ kg$$

(65)

→ $g = \frac{Gm}{R^2} = \frac{G4\pi R^3\rho}{3R^2}$

$$= \frac{4}{3}\pi G\rho R$$

$$\therefore \rho \propto g$$

(66)

→ $g = \frac{GM}{R^2}$ ________________ (1)

→ Density $d = \frac{M}{V} = \frac{M}{\left(\frac{4}{3}\pi R^3\right)}$

$$\therefore\ M = \left(\frac{4}{3}\pi R^3\right)d.$$

$$\therefore\ g = G\frac{\left(\frac{4}{3}\pi R^3\right)}{R^2}d.$$

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$= G\left(\frac{4}{3}\pi R\rho\right)$

→ $g \propto R$

$\frac{g'}{g} = \frac{R'}{R}$

$\therefore \frac{g'}{g} = \frac{3R}{R}$

$\therefore g' = 3g$

(76)

$\left(\frac{dg}{g}\right) \times 100\% = -2\left(\frac{dr}{r} \times 100\%\right)$

$= -2\ (-0.5\%)$

$= +1\%$

Value of g increases by 1%.

(77)

⇒ $g \propto \frac{1}{r^2} \quad \therefore \frac{g_2}{g_1} = \left(\frac{r_1}{r_2}\right)^2$

But $r_1 = R_e$ and $r_e = R_e + 2R_e = 3R_e$

$\therefore \frac{g_2}{g_1} = \left(\frac{R_e}{3R_e}\right)^2 = \frac{1}{9}$

$\therefore g_2 = 9.81 / 9$ [$\because g_1 = g = 9.81\ ms^{-2}$]

$\therefore g_2 = 1.09\ m/s^2$

(86)

→ $h = 1m$

→ $g_h = g\left(1 - \frac{2h}{R}\right)$

→ $g_d = g\left(1 - \frac{d}{R}\right)$

→ $g_h = g_d$

$g\left(1 - \frac{d}{R}\right) = g\left(1 - \frac{2h}{R}\right)$

→ $d = 2h$

(88)

→ Gravitational acceleration on earth's surface,

$g_e = \frac{GM_e}{R^2}$

Gravitational acceleration at a height h from surface of earth,

$g = \frac{GM_e}{(R+h)^2}$

$\therefore \frac{g_e}{g} = \frac{(R+h)^2}{R^2}$

$\therefore 2 = \frac{(R+h)^2}{R^2}$

$\therefore \sqrt{2} = \frac{R+h}{R}$

$\therefore R(\sqrt{2} - 1) = h$

(89)

→ $\frac{W_1}{W_2} = \frac{mg}{mg'} = \frac{g}{g'}$

→ $\frac{g_1}{g_2} = \frac{Gm_e}{R_e^2}\frac{(R_e + h)^2}{GM_e}$

→ $\frac{W_1}{W_2} = \left(\frac{R_e + h}{R_e}\right)^2$

$\frac{500}{250} = \left(1 + \frac{h}{R_e}\right)$

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→ $\frac{h}{R_e} = \sqrt{2} - 1$

→ h = 6400 × 0.414

h = 2650 Km

(90)

→ mg = F = 72 N. R (weight on the surface)

$g = \frac{GM}{R^2}$

At height, H = R/2

$\therefore\ g' = \frac{4}{9}\frac{GM}{R^2}$

$\therefore\ mg' = m\left(\frac{4}{9}\frac{GM}{R^2}\right)$

$= \frac{4}{9}(mg)$

$= \frac{4}{9}(72)$

= 32 N

(93)

For weightlessness

$W = \sqrt{\frac{g}{R_e}}$

→ $W = 1.56 \times 10^{-3}$

(94)

When effective g becomes zero, object become weightless.

$g' = g - \omega^2 R\cos^2\lambda$

$0 = g - \omega^2 R\cos^2(45^o)$

$g = \omega^2 R \times \frac{1}{2}$

$\omega = \sqrt{\frac{2g}{R}}$

(97)

From, $g' = g - \omega^2 R_e$

If rotation stops $\omega = 0$

The value of g increases by $\omega^2 R_e$.

NOTE

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6 Gravitational Intensity :

→ The gravitational force on a body by the other one is given by Newton's law of gravitation.

$$F_{12} = \frac{G\, m_1\, m_2}{r^2}$$

→ This process of action of force exerted mutually on two bodies separated by some distance is explained to occur through the field as under :

(1) Every object produces a **gravitational field** around it, due to its mass.

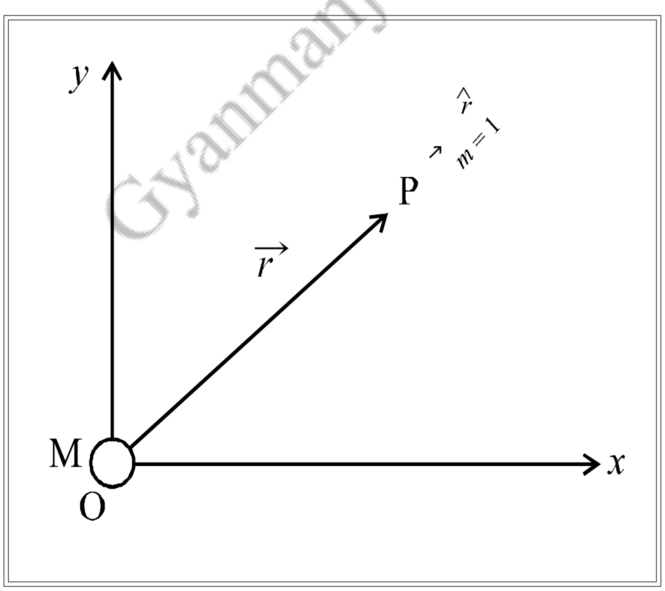
(2) This field exerts a force on another body brought (or lying) in this field.Hence it is important to know about the strength of such a gravitational field.

→ **"The gravitational force exerted by the given body on a body of unit mass at a given point is called the intensity of gravitational field (I) at that point."**

Direction : In direction of force, I is function of position $\vec{I} = \vec{F} / m$

- It is also known as the **gravitational field or gravitational intensity.**

→ Using Newton's law of gravitation we can write the formula for the gravitational intensity. Consider a body of mass M at the origin of co-ordinate system O.



→ The gravitational intensity ($\vec{I}$) due to it at some point P is,

$$F = \frac{GMm}{r^2}$$

$$\therefore\ I = \frac{F}{m} = \frac{GM}{r^2}$$

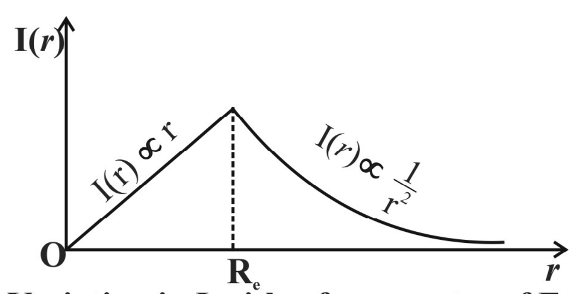
- In magnitude we can write $\boxed{I = \frac{GM}{r^2}}$ —(1)
  Remember
- Its unit is N/kg or $\boxed{m/s^2}$.
- the dimensional formula is $M^0L^1T^{-2}$.

→ Now if a body of mass m is put (or lying) at this point P, the gravitational force exerted by the field on it is

$\boxed{\vec{F} = \vec{I}\, m}$ —(2)

If I is given then use directly F = I m

→ Equation (1) shows that the gravitational intensity due to earth at a point has the same value as the gravitational accleration at that point.(♠) But these two quantities are different and their units are different but equivalent. [N/kg = m/s^2]. It is obvious that I – r graph for the Earth's gravitational field would be same as g – r graph like following figure.



Variation in I with *r* from centre of Earth

(♠) $I = GM_e / r^2$ (Intensity at distance r due to earth)

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::: Short Explanation :::

➥ **Gravitational intensity (I) :**

→ Every mass m produces gravitational field around it.

→ This gravitational field intensity is given by the equation $\vec{I} = \frac{\vec{F}}{m}$.

Means, the intensity of the gravitational field of a material body at any point in its field is defined as the gravitational force experienced by a unit mass placed at that point.

→ $I = \frac{GM}{r^2}$ (❑)

Actually, $I = -GM/r^2$ due to attractional force. Hence I → r graph is in 4th quadrant. See sloved pro. 99. But in option if graph is not given in 4th quadrant, then considered above graph which represent only magnitude.

where

M = magnitude of mass which produces gravitational field intensity.

r = distance between a point where we want to find I and the mass that produces gravitational field

→ Direction of $\vec{I}$ = direction of $\vec{F}$

∴ Direction of $\vec{I}$ is in the direction of mass M

→ $\vec{I} = \frac{\vec{F}}{M}$ so it is also termed as gravitational acceleration g.

∴ I produced due to earth is,

$I = g = \frac{g_e}{R_e} r$ (for $r \le R_e$)

$I = g_e = \frac{GM_e}{R_e^2}$ (for $r = R_e$)

and $I = g = \frac{GM_e}{r^2}$ (for $r \ge R_e$)

(❑) $I = (GM_e / r^2)$ is intensity due to earth at distance r.

7 Gravitational Potential and Gravitational Potential Energy in the Earth's Gravitational Field :

7.1 (a) Gravitational Potential :

→ Every object produces a gravitational field around it. A characteristic of such a field is defined as a quantity called the gravitational potential as under :

7.1.1 **"The negative of the work done by the gravitational force in bringing a unit mass from infinite distance to the given point in the gravitatinal field is called gravitational potential (ϕ) at that point."**

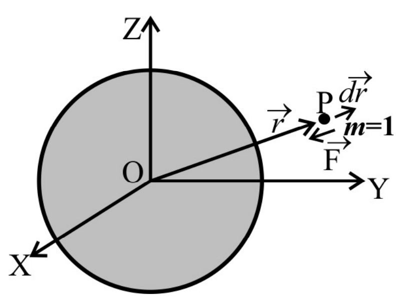
scaler quantity

- The unit of gravitational potential is J kg^{-1}

square of unit of velocity (m^2/s^2)

- dimensional formula is $M^0L^2T^{-2}$.

7.1.2 Formula



$\vec{dr} = dr\,\hat{r}$

→ We put the origin of the co-ordinate system at the centre of the Earth.

- Mass of the Earth is M_e
- Radius is R_e.
- The position vector of point P at distance r from the center of the Earth is $\vec{OP} = \vec{r}$. Here, $r \ge R_e$.

→ At this point the Earth's gravitational force on a body of **unit mass** is

$$\vec{F} = \frac{-GM_e(1)}{r^2}\hat{r}$$

$$\vec{F} = \frac{-GM_e}{r^2}\hat{r} \quad \text{——(1)}$$

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→ This force is not constant but changes with distance. But during an infinitely small displacement $d\vec{r}$ the force can be taken as constant. Hence, during such a small displacement, the work done by the gravitational force is

$$dW = \vec{F} \cdot d\vec{r} = \left(\frac{-GM_e}{r^2}\hat{r}\right) \cdot (dr\,\hat{r}) \text{ —— (2)}$$

$$dW = \frac{-GM_e}{r^2}\,dr \text{ —— (3)}$$

→ The entire path from point P to infinite distance can be divided in large number of infinitely small intervals.

→ Taking the force as constant during every such interval, we can calculate the work done during that interval, and by adding all such works we get the total work W.

→ As this process is a continuous one, the summation can be written as integration.

→ Hence, in this case, work done by the gravitational force in moving this body from point P at distance r to infinite distance is

$$W_{r\to\infty} = \int_r^\infty dW = \int_r^\infty \left(-\frac{GM_e}{r^2}\right) dr$$

$$= -GM_e \int_r^\infty \frac{1}{r^2}\,dr$$

$$= -GM_e\left[-\frac{1}{r}\right]_r^\infty = +GM_e\left[\frac{1}{\infty} - \frac{1}{r}\right]$$

$$\boxed{W_{r\to\infty} = \frac{-GM_e}{r}} \quad \boxed{\frac{r^{-2+1}}{-2+1} = \frac{r^{-1}}{-1} = \frac{-1}{r}} \text{ —— (4)}$$

→ Now, if we bring this body from infinite distance to the point P at distance r; the work ($W_{\infty\to r}$) **by the gravitational force** will be the same as that given by equation (4) but with opposite sign.

→ [$W_{\infty\to r} = -W_{r\to\infty}$], because gravitational force is a conservative force.

$$\boxed{\therefore W_{\infty\to r} = \frac{GM_e}{r}} \text{ —— (5)}$$

In a closed path total work will be zero

The negative of this work ($W_{\infty\to r}$) is by definition, called the gravitational potential ϕ at point P,

∴ gravitational potential at P is,

$$\boxed{\phi = \frac{-GM_e}{r} = -gr} \text{ —— (6)}$$

each mass particle produce gr. field and gr. potential around it.

→ From this the gravitational potential at the Earth's surface (putting $r = R_e$) is,

$$\boxed{\phi_e = \frac{-GM_e}{R_e}} \text{ —— (7)}$$

$= -0.63 \times 10^8$ J/kg also becomes $-g_e R_e$

7.1.3 We note a few points about the gravitational potential :

(1) The gravitational potential at infinite distance from the center of the Earth = 0

In equation (6) by taking $r = \infty$, ϕ will be zero

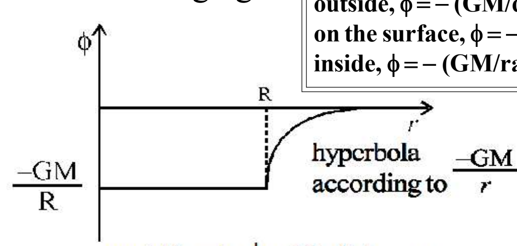
(2) **The graviatational potential at all points inside** a uniform spherical shell **is the same,** and is equal to the value at the surface that is, equal to –GM/R. where, M = mass of the shell and R = radius of the shell.

In Std. -12, Ch.-1,2 discussion of electric field & electric potential produce due to electric charge, you will derive different formula on different shape of electric charge particle for distribution and you will get I & ϕ for gravition.

→ The reason for this is that the gravitational force at all points inside the shell is zero, hence **no work is done in the motion of the body inside the shell.** The work during the motion from infinite distance to surface only comes in the calculation.

(3) The variation in the gravitational potential with distance r from the centre of the shell, having mass M and radius R is shown in following figure

For shell :
outside, $\phi = -$ (GM/distance)
on the surface, $\phi = -$ (GM/radius)
inside, $\phi = -$ (GM/radius)



variation in ϕ with distance r

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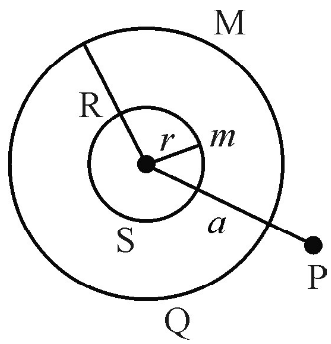
★ **Extra MCQ**

For shell

$\phi_P = \frac{-Gm}{a} + \frac{-GM}{a}$

$\phi_Q = \frac{-Gm}{R} + \frac{-GM}{R}$

$\phi_S = \frac{-Gm}{r} + \frac{-GM}{R}$



7.2 (b) Gravitational Potential Energy

→ **"The negative of the work done by the gravitational force in bringing a given body (of mass m) from infinite distance to the given point in the gravitational field is called the gravitational potential energy U of that body at that point."** (Scaler quantity)

- It is actually the gravitational potential energy of the system of the Earth + that body.

→ Considering definitions of gravitational potential, gravitational potential energy and using equation $\boxed{\phi = \frac{-GM_e}{r}}$ ——— (1)

the gravitational potential energy of a body of mass m at a distance r from the Earth's center ($r \geq R_e$) is,

$\boxed{U = \phi m = \frac{-GM_e m}{r}}$ ——— (2)

→ Hence the gravitational potential energy of the body of mass m, lying on the surface of the Earth ($r = R_e$) is,

$\boxed{U_e = \frac{-GM_e m}{R_e}}$ $\boxed{= -mg_e R_e}$ ——— (3)

→ We can also say that the gravitational potential is the gravitational potential energy of unit mass.

→ At infinite distance from the centre of the Earth the gravitational force of the Earth on that body is zero and according to the above definition we can say that its gravitational potential energy is also zero. ($r = \infty \Rightarrow U = 0$)

→ The **absolute value** of the potential energy (or potential) has **no importance** at all, only the **change** in its value **is important**. Hence the reference point for zero potential energy (or zero potential) can be taken anywhere.(✦)

→ Here the potential energy U is of the system consisting of the Earth and the body. But in this process the position of Earth or its velocity is not appreciably changed, hence it is also conventionally mentioned as the potential energy **of the body.** Whenever such a mention is made we have to understand that this potential energy is actually **of that system** but the entire change in that potential energy appears to be experienced by the **body alone.(♠)**

Note : In future, we are also going to consider a satellite. In that case the potential energy is of the system consisting of the Earth and the satellite. But we shall mention it as potential energy of the satellite.

(✦) **You may recall** that in the chapter of 'Work Energy and Power' we had taken zero potential energy at the surface of the Earth, while here we have taken zero potential energy at infinite distance. But in both the cases only the changes are important hence no contradiction is produced.

(♠) **(1)** If the distance between m_1 and m_2 is r_{12}, then gravitational potential energy $U_{12} = -Gm_1m_2/r_{12}$.

(2) If the system is madeup of 5 particles
$U = U_{12} + U_{13} + U_{14} + U_{15} + U_{23} + U_{24} + U_{25} + U_{34} + U_{35} + U_{45}$
= Do the sum of potential energy of all pairs in the system.

(3) To bright an object of mass m to height of nR_e from earth's surface change in gravitational potential energy $\Delta U = mgR_e\,[(\,n\,/\,(\,n+1\,)]$.

See Solved problem : 112 to 114, 38, 2

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➥ (*b*) Gravitational potential energy :

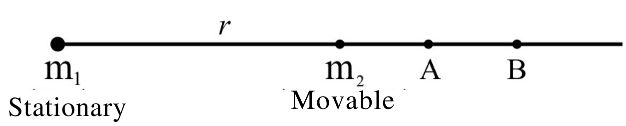
→ Gravitational potential energy due to a conservative forces of any system is defined as,

$$U_f - U_i = -W = -\int_i^f \vec{F} \cdot d\vec{r}$$

i.e. change in potential energy

= negative amount of work done by internal conservative force.

➥ Illustration : (gravitatioan potential energy)



→ Internal conservative force (gravitational force) from above figure is,

$$F = \frac{Gm_1m_2}{r^2} \qquad (\text{ towards } m_1 \text{ })$$

→ Now, if m_2 is moved slightly from A to B, then small amount of work done by gravitational force,

$$dW = -\frac{Gm_1m_2}{r^2}\, dr$$

∴ So the increase in potential energy of the system of two body, during the movement, is

$$dU = -dW = \frac{G\, m_1\, m_2}{r^2} dr$$

→ Now if the distance between two particles is changed from r_1 to r_2 then change in potential energy,

$$U(r_2) - U(r_1) = \int dU = \int_{r_1}^{r_2} \frac{G\, m_1\, m_2}{r^2}\, dr$$

$$= G\, m_1 m_2 \left[-\frac{1}{r} \right]_{r_1}^{r_2}$$

$$U(r_2) - U(r_1) = G\, m_1\, m_2 \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

In gravitational field of mass M , a particle of mass m is moving from r_1 and r_2 distance (where r_1 & r_2 are distance from centre of mass M) then change in gravitational potenttial energy $\Delta U = GMm\, [\, (1/r_1) - (1/r_2)\,]$

→ If zero of potential energy is taken when two particles are at infinite distance then, change is potential when they are r distance apart is,

$$U(r) - U(\infty) = G\, m_1\, m_2 \left(\frac{1}{\infty} - \frac{1}{r} \right) = -\frac{Gm_1\, m_2}{r}$$

∴ the gravitational potential energy of a system of two particles placed at a distance r, is

$$U(r) = -\frac{G\, m_1\, m_2}{r}.$$

→ When particles are closer, potential energy decreases and when they go away from each other, potential energy increases. (The work done by external force which doesn't result into kinetic energy always converted into potential energy)

→ If system comprises of more than two particles then add the potential energy of all possible pairs.

Notes

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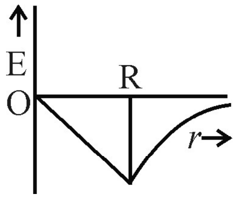
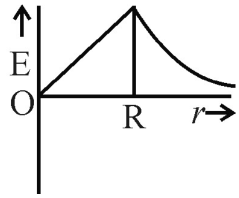
Solved Problems

(99) Dependence of intensity of gravitational field(E) of earth with distance(r) from centre of earth is correctly represented by [PMT : 2014]

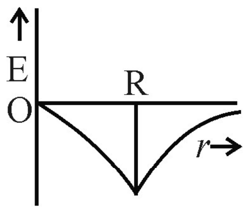
(A)



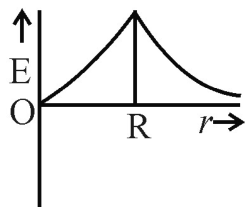
(B)



(C)

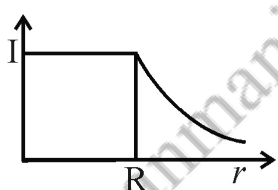


(D)

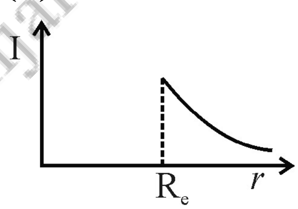


(100) Which of the following graph shows the variation in the gravitational intensity I with distance from the centre of a spherical shell with uniform density and radius R.

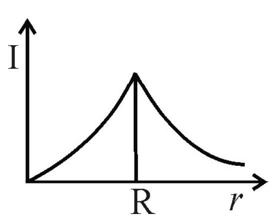
(A)



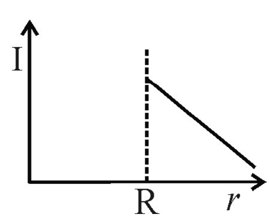
(B)



(C)



(D)



Ans : (99) A (100) B (102) C

(101) The gravitational intensity at a point is $\vec{I} = 10^{-9}(\hat{i}+\hat{j})$ N/kg. If a body of 10 kg mass is placed at this point, find the magnitude of force on it and the magnitude of its acceleration.

(★) Obtained T for $\lambda = 45^0$ lattitude

Answer will be, $T = 2\pi\sqrt{\frac{R_e}{g}}$

Solution:

$\rightarrow \vec{F} = (\vec{I})(m) = (10^{-9})(\hat{i}+\hat{j})(10)$

$= 10^{-8}\hat{i} + 10^{-8}\hat{j}$ N

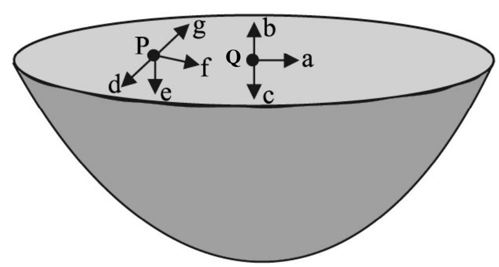
$\therefore |\vec{F}| = \sqrt{(10^{-8})^2 + (10^{-8})^2}$

$= 10^{-8}\sqrt{2} = 1.414 \times 10^{-8}$ N

$\rightarrow g = \frac{|\vec{F}|}{m} = \frac{1.414 \times 10^{-8}}{10} = 1.414 \times 10^{-9}$ m/s²

(102) The gravitational intensity at the centre of a hemispherical shell of uniform mass density has the direction indicated by the arrow (see Figure)

(i) a, (ii) b, (iii) c, (iv) 0.



Solution :

⇒ If the shell is completely spherical intensity inside the shell is zero at every point.

So intensity due to lower hemispherical shell, is in downward direction and intensity due to upper hemispherical shell is in upward direction. So resultant intensity inside the shell becomes zero.

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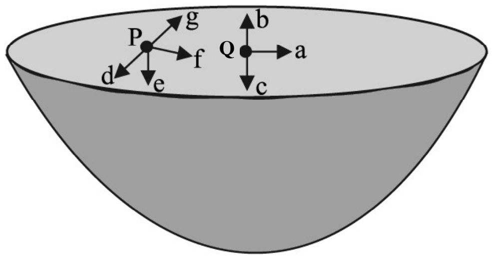
(103) For the above problem, the direction of the gravitational intensity at an arbitrary point P is indicated by the arrow (i) d, (ii) e, (iii) f, (iv) g.



(104) The magnitudes of the gravitational field at distance r_1 and r_2 from the centre of a uniform sphere of radius R and mass m are F_1 and F_2 respecitvely. Then... [IIT : 1994]

(A) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ **if $r_1 < R$ and $r_2 < R$**

(B) $\frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$ **if $r_1 > R$ and $r_2 > R$**

(C) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ **if $r_1 > R$ and $r_2 > R$**

(D) $\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2}$ **if $r_1 < R$ and $r_2 < R$**

Solution : [Ans. (A,B)]

(105) If the gravitational potential at the Earth's surface is ϕ_e, what is the gravitational potential at a height from Earth's surface equal to its radius?

(A) $\phi_e / 2$ **(B)** $\phi_e / 4$

(C) ϕ_e **(D)** $\phi_e / 3$

Solution : [Ans. (A)]

$\Rightarrow$ gravitational potential at distance r from the centre of the Earth.

in $\phi = -\frac{GM_e}{r}$, equal $(-GM_e)$

$\therefore \ \phi \propto \frac{1}{r}$

$\therefore$ in $\frac{\phi_2}{\phi_1} = \frac{r_1}{r_2}$, $r_1 = R_e \Rightarrow \phi_1 = \phi_e$ and $r_2 = 2R_e$

$\therefore \ \phi_2 = \phi_e \times \frac{R_e}{2R_e} = \frac{\phi_e}{2}$

(106) If the Earth shrinks (but not cut !) in such a way that its radius becomes (R_e/2) from R_e, what can we say about the values of gravitational potential ϕ, at a point at distance R_e from its centre in the two cases ?

(A) the values of g and ϕ both become half.

(B) the value of g becomes half and the value of ϕ remains the same as before.

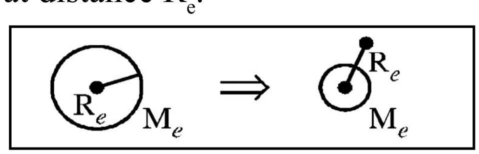
(C) the value of g remains the same as before and the value of ϕ becomes half.

(D) the values of g and ϕ both remain the same as before.

Solution : [Ans. (D)]

$\Rightarrow$ Here, $g = GM / r^2$ and $\phi = GM / r$. M, G and $r = R_e$ distance are same, so the value of g and ϕ remains constant does not changes.

Here, it is not asked for surface, but asked at distance R_e.



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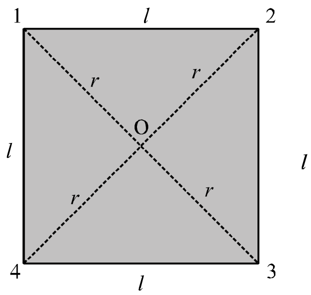
(107) A particle of mass m is placed on each vertex of a square of side l as shown in figure. Calculate the gravitational potential energy of this system of four particles. Also calculate the gravitational potential at the center of the square. (☞)



Ans :

→ Here we can write the potential energy due to every pair of particles as

$U_{ij} = \frac{-Gm_i m_j}{r_{ij}}$, where m_i and m_j are the masses of particles i and j respectively, and r_{ij} is the distance between them.

$m_i = m_j = m$.

∴ Total potential energy

$$U = U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34} \quad (☆)$$

$$= -Gm^2 \left(\frac{1}{r_{12}} + \frac{1}{r_{13}} + \frac{1}{r_{14}} + \frac{1}{r_{23}} + \frac{1}{r_{24}} + \frac{1}{r_{34}} \right)$$

$$= -Gm^2 \left(\frac{1}{l} + \frac{1}{\sqrt{2}l} + \frac{1}{l} + \frac{1}{l} + \frac{1}{\sqrt{2}l} + \frac{1}{l} \right)$$

$$= -Gm^2 \left(\frac{4+\sqrt{2}}{l} \right)$$

Here, $r_{13} = r_{24} = \sqrt{2}\, l$

→ $r_{12} = r_{14} = r_{23} = r_{34} = r = l$

(☞) If n particles are in system then, no. of pair will be $\binom{n}{2} = \frac{n(n-1)}{2}$

(☆) Add P.E. for each pair containing in the systems.

→ The gravitational potential at the center, due to each particle is same.

∴ The total gravitational potential at the center of the square is

ϕ = 4 (potential due to every particle)

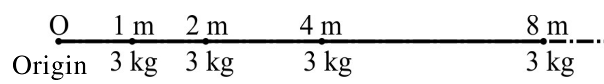
$$= 4 \left(\frac{-Gm}{r} \right)$$

$$\boxed{\phi = \frac{-4\sqrt{2}\, Gm}{l}} \quad \left(\because \text{by putting } r = l/\sqrt{2} \right)$$

(108) Each object of mass 3 kg is placed on x-axis at distances 1m, 2m, 4m, 8m, ... upto ∞ from the origin of coordinate system. Find the gravitational intensity at the origin in terms of the gravitational constant G.

[Form a similar example for gravitational potential by yourself and solve it. See CTP-121, P.no. - 49]

Solution :



From fig. gravitational intensity at point O,

$$I = \frac{Gm}{r_1^2} + \frac{Gm}{r_2^2} + \frac{Gm}{r_3^2} + \frac{Gm}{r_4^2} + \dots$$

$$= \frac{Gm}{1^2} + \frac{Gm}{2^2} + \frac{Gm}{4^2} + \frac{Gm}{8^2} + \dots$$

$$= \frac{G(3)}{1} + \frac{G(3)}{4} + \frac{G(3)}{16} + \frac{G(3)}{64} + \dots$$

$$= 3G \left[\frac{1}{1} + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots \right]$$

$S_n = \frac{a}{1-r}$. Here $a = 1, r = \frac{1}{4}$

$$\therefore I = 3G \left[\frac{1}{1-\frac{1}{4}} \right] = 3G \left[\frac{4}{3} \right] \qquad \therefore I = 4G$$

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(109) At points inside a uniform spherical shell........

(A) gravitational intensity and gravitational potential both are zero

(B) gravitational intensity and gravitational potential both are non-zero

(C) gravitational intensity is non zero and gravitational potential is zero

(D) gravitational intensity is zero and gravitational potential is non-zero

Solution : [Ans. (D)]

(110) A particle of mass M is situated at the centre of a spherical shell of same mass and radius *a*. The gravitational potential at a point situated at *a*/2 distance from the centre, will be .... (Take mass of shell also M)

[NEET : 2010]

(A) $-\dfrac{3GM}{a}$ (B) $-\dfrac{2GM}{a}$

(C) $-\dfrac{GM}{a}$ (D) $-\dfrac{4GM}{a}$

Solution :

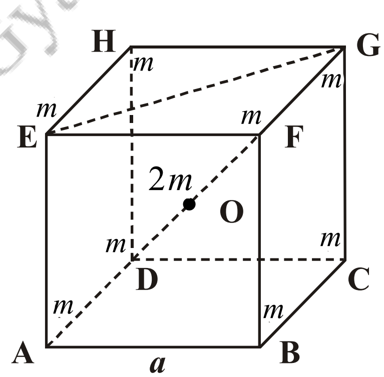
Potential at the given point

= potential due to shell

+ potential due to particle.

$$= \frac{-GM}{a} - \frac{2GM}{a} = \frac{-3GM}{a}$$

(111) Calculate the gravitational potential and gravitational potential energy at the centre of the cube made up of 9 masses , shown in the figure.



Answer : (110) A (112) C

Solution :

→ Potential energy of 12 pairs like AB

$$U_1 = 12\left(\frac{-Gm^2}{a}\right) \text{———} (1)$$

→ Potential energy of 12 pairs like AF

$$U_2 = 12\left(\frac{-Gm^2}{\sqrt{2}a}\right) \text{———} (2)$$

→ Potential energy of 4 pairs like AG

$$U_3 = 4\left(\frac{-Gm^2}{\sqrt{3}a}\right) \text{———} (3)$$

→ Potential energy of 8 pairs like AO

$$U_4 = 8\left(\frac{-G(m)(2m)}{(\sqrt{3}/2)a}\right) \text{———} (4)$$

→ Total potential energy of a system.

$$U = U_1 + U_2 + U_3 + U_4$$

$$U = \frac{-Gm^2}{a}\left[12\left(1 + \frac{1}{\sqrt{2}}\right) + \frac{36}{\sqrt{3}}\right]$$

$$\phi = -\frac{16}{\sqrt{3}}\frac{Gm}{a}$$

(112) If a body of mass m on the Earth's surface is taken to a height equal to nR from the surface; what would be the change in its potential energy ? (R = radius of the earth g = gravitational acceleration at the earth's surface.)

(A) $\dfrac{mgR}{(n-1)}$ **(B)** $mgR\,(n-1)$

(C) $mgR\dfrac{n}{n+1}$ **(D)** $mgR\,(n+1)$

Solution :

Potential energy at surface of earth

$$U_1 = \frac{-GMm}{R}$$

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Potential energy at height nR from surface of earth

$$U_2 = \frac{-GMm}{R+nR}$$

$\Delta U = GMm\,[(1/r_1) - (1/r_2)]$ can be directly used.

$$= \frac{-GMm}{(n+1)R}$$

Change in potential energy $U_2 - U_1$

$$= \frac{-GMm}{(n+1)R} - \left(\frac{GMm}{R}\right)$$

$$= \frac{GMm}{R}\left(1 - \frac{1}{n+1}\right)$$

$$= \frac{GMm}{R}\left(\frac{n}{n+1}\right)$$

$$= gmR\left(\frac{n}{n+1}\right) \qquad \left(\because \frac{GM}{R} = gR\right)$$

(113) If 'g' is the acceleration due to gravity on the earth's surface., the gain in the potential energy of an object of mass 'm' raised from the surface of the earth to a height equal to the radius 'R' of the earth is .... [AIEEE : 2004]

(A) $\frac{1}{4}$ *mgR* **(B) $\frac{1}{2}$ *mgR***

(C) *2mgR* **(D) *mgR***

Solution :

$$\Delta U = U_2 - U_1$$

$$\Delta U = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\Delta U = GMm\left(\frac{1}{R} - \frac{1}{2R}\right) = \frac{GMm}{2R}$$

$$\Delta U = \frac{1}{2}\, mgR \qquad \left(\because g = \frac{Gm}{R^2}\right)$$

Answer : (113) B (114) C (115) D

(114) A body of mass m is placed on earth's surface. It is then taken from earth's surface to a height h = 3R, then the change in gravitational potential energy is.... [2002]

(A) $\frac{mgh}{R}$ **(B) $\frac{2}{3}\, mgR$**

(C) $\frac{3}{4}\, mgR$ **(D) $\frac{mgR}{2}$**

Solution :

$$\Delta U = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Where $r_1 = R$, $r_2 = 4R$

$$\Delta U = GMm\left(\frac{1}{R} - \frac{1}{4R}\right)$$

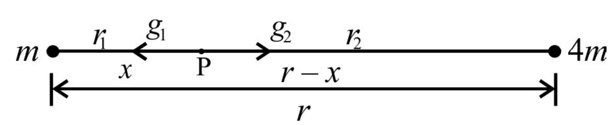
$$= \frac{3}{4}\frac{GMm}{R} = \frac{3}{4}\, mgR \qquad \left(\because g = \frac{GM}{R^2}\right)$$

(115) Two objects of mass m and 4m are placed at distance r. What is the gravitational potential on the line joining of these objects at a point where gravitational field is zero. [AIEEE : 2011]

(A) zero **(B) $\frac{-4Gm}{r}$**

(C) $\frac{-GMm}{r}$ **(D) $\frac{-9Gm}{r}$**

Solution :



→ At point P resultant gravitational field becomes zero.

for that $g_1 = g_2$

$$\frac{Gm_1}{r_1^2} = \frac{Gm_2}{r_2^2}$$

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$$\frac{m}{x^2} = \frac{4m}{(r-x)^2}$$

$$(r-x)^2 = 4x^2$$

$$r - x = 2x$$

$$\therefore x = r/3$$

$$\therefore r_1 = r/3 \text{ or } r_2 = \frac{2r}{3}$$

Total gravitational potential at point P

$$\phi_p = \left(\frac{-Gm_1}{r_1}\right) + \left(\frac{-Gm_2}{r_2}\right)$$

$$= \frac{-Gm}{r/3} - \frac{G(4m)}{2r/3}$$

$$= \frac{-3Gm}{r} - \frac{6\,Gm}{r}$$

$$\therefore \phi_p = -9\frac{Gm}{r}$$

(116) At what height from the surface of earth the gravitation potential and the value of g are – 5.4 × 10^7 J kg^{-2} and 6.0 ms^{-2} respectively ? Take the radius of earth as 6400 km : [NEET : 2016]

(A) 1600 km (B) 1400 km

(C) 2000 km (D) 2600 km

Solution :

Gravitational potential

$\phi = -5.4 \times 10^7$ J kg^{-2}

Gravitational Acceleration g = 6 ms^{-2}

Gravitational potential at some height h from the surface of the earth,

$$\phi = -\frac{GM}{R+h} \qquad ....(i)$$

and acceleration due to gravity at some height h from the surface of the earth.

$$g = \frac{GM}{(R+h)^2} \qquad ....(ii)$$

From Equation (i) and (ii),

$$\frac{\phi}{g} = R + h$$

$$\therefore \frac{5.4 \times 10^7}{6} = R + h$$

→ h = 2600 km

(117) Choose any one of the following four responses :

(A) If both Assertion and Reason are true and reason is the correct explanation of the Assertion.

(B) If both Assertion and Reason are true but Reason is not a correct explanation of the Assertion.

(C) If Assertion is true but Reason is false

(D) If both Assertion and Reason are false.

<u>Assertion</u> :

By taking a body away from the Earth the potential energy of the system of Earth and that body increases.

<u>Reason</u> :

To increase the separation between them, work has to be done by applying external force against the attractive force between them.

(A) A (B) B (C) C (D) D

Answer : (117) A

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(118) An object of mass m placed at a point B in the gravitational field of mass M. If this object is taken from point B and moving towards point A in the gravitational filed then gravitational potential energy is ....

(A) remains constant

(B) increases

(C) decreases

(D) becomes zero

Solution :

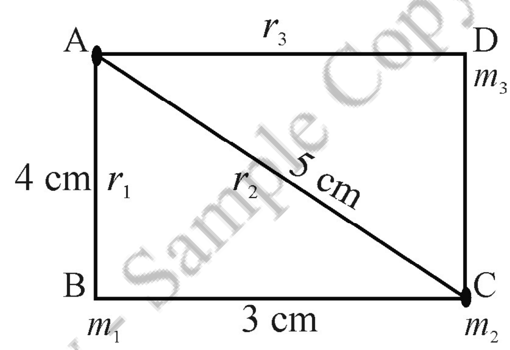
In the attractive field, potential energy decreases.

Notes

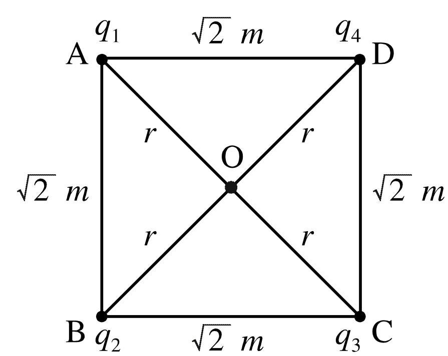
Answer : (118) C

Current Topic Practice

(119) 10 kg, 20 kg and 10 kg point masses are placed on the respective vertices B, C and D of a ractengle. If AB = 4 cm and BC = 3 cm then calculate the gravitational potential at the vertex A. $G = 6.67 \times 10^{-11}\ Nm^2 / kg^2$.



(120) 10 kg, 10 kg, 20 kg and 30 kg masses are placed on the respective vertices A, B, C and D of a square ABCD having length of its side $\sqrt{2}$ m. Calculate the gravitational potential and gravitational potential energy at the center of the square. $G = 6.67 \times 10^{-11}\ Nm^2 / kg^2$



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(121) From the origin of a co-ordinate system a mass m is placed at each point with x =1, 2, 4, 8, 16, ... (upto infinity) distances, what is the gravitational potential at the origin ?

(A) – 2 Gm (B) – 4 Gm

(C) – 2 / Gm (D) – 4 / Gm

(122) Which of the following alternatives represents the dimensional formula of the gravitational potential and gravitational potential energy respectively?

(A) $M^1L^1T^{-1}$, $M^1L^2T^{-2}$

(B) $M^0L^2T^{-2}$, $M^1L^2T^{-2}$

(C) $M^0L^2T^{-2}$, $M^1L^2T^2$

(D) $M^1L^2T^{-1}$, $M^2L^1T^{-1}$

(123) A body of mass 'm' is taken from the earth's surface to the height equal to twice the radius (R) of the earth. The change in potential energy of body will be :

[NEET : 2013]

(A) –3 mgR (B) –1/3 mgR

(C) 2 mgR (D) 2/3 mgR

(124) A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them to take the particle far away from the sphere

(Take $G = 6.67 \times 10^{-11}$ Nm^2/kg^2)

[AIEEE : 2005]

(A) 3.33×10^{-10} J (B) 13.34×10^{-10} J

(C) 6.67×10^{-10} J (D) 6.67×10^{-9} J

(125) Infinite number of bodies, each of mass 2 kg are situated on x-axis at distance 1m, 2m, 4m, 8m, ... , respectively, from the origin. The resulting gravitational potential due to this system at the origin will be .... [NEET : 2013]

(A) $-\frac{4}{3}$G (B) – 4G

(C) – G (D) $-\frac{8}{3}$G

(126) Energy required to move a body of mass m from an orbit of radius 2R to 3R is

[AIEEE : 2002]

(A) GMm / 12R^2 (B) GMm / 3R^2

(C) GMm / 8R (D) GMm / 6R

(127) As you have learnt in the text, a geostationary satellite orbits the earth at a height of nearly 36,000 km from the surface of the earth. What is the potential due to earth's gravity at the site of this satellite ?

(Take the potential energy at infinity to be zero).

Mass of the earth = 6.0×10^{24} kg,

radius = 6400 km.

(128) Two heavy spheres each of mass 100 kg and radius 0.10 m are placed 1.0 m apart on a horizontal table. What is the gravitational force and potential at the mid point of the line joining the centres of the spheres ? Is an object placed at that point in equilibrium? If so, is the equilibrium stable or unstable ?

***Ans* : (121) A (122) B (123) D (124) C (125) B (126) D**

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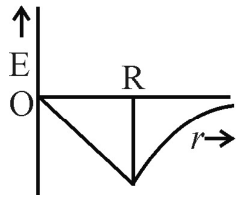
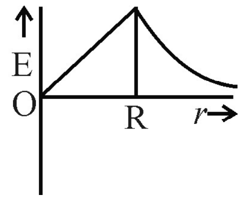
(129) Dependence of intensity of gravitational field(E) of earth with distance(r) from centre of earth is correctly represented by [NEET : 2014]

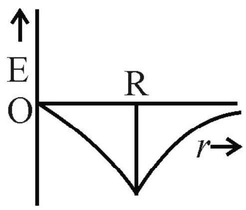
(A)



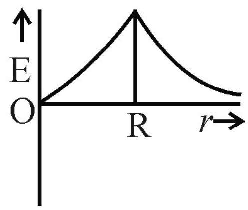
(B)



(C)



(D)



Notes

Hints & Solutions

(119)

$$\phi = -G\left(\frac{m_1}{r_1} + \frac{m_2}{r_2} + \frac{m_3}{r_3}\right)$$

(120)

$$\phi = -G\left(\frac{m_1}{r} + \frac{m_2}{r} + \frac{m_3}{r} + \frac{m_4}{r}\right)$$

(121)

$\rightarrow$ $\phi = -Gm\left[\frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots\right]$

$$S = \frac{a}{1-r}$$

$$\phi = -Gm\left[\frac{1}{1-½}\right]$$

$$\phi = -2Gm$$

(122)

$\Rightarrow$ Gravitational potential ϕ = Energy/Mass

$\therefore$ $[\phi] = [E] / [M] = M^1L^2T^{-2} / M^1$

$= M^0L^2T^{-2}$

and gravitational potential Energy $U = \phi_m$

$= (M^0L^2T^{-2})(M^1L^0T^0) = M^1L^2T^{-2}$

(123)

$$\Delta U = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Where $r_1 = R$

$r_2 = 3R$

$$\Delta U = \frac{2}{3} m g R \qquad \left(\because g = \frac{GM}{R^2}\right)$$

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(124)

Work done = Potential Energy of System

$$= \frac{G\,M\,m}{r}$$

$$= \frac{6.67 \times 10^{-11} \times 100 \times 10 \times 10^{-3}}{10 \times 10^{-2}}$$

$W = 6.67 \times 10^{-10}$ J

(125)

Gravitational Potential $\phi = \frac{-GM}{R}$

The resulting gravitational potential,

$$V = -2G\left[\frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...\right]$$

$$= -2G\left[1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + ...\right]$$

$$= -2G\left[1 + \frac{1}{2}\right]^{-1}$$

$$= \frac{-2G}{\left(1-\frac{1}{2}\right)} = \frac{-2G}{1/2} = -4G$$

(126) Energy required to increase the orbital radius of satellite is,

$$\Delta U = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = GMm\left(\frac{1}{2R} - \frac{1}{3R}\right)$$

$$\Delta U = \frac{GMm}{6R}$$

(127)

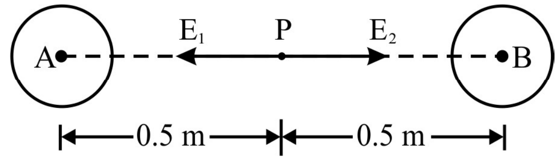
Gravitational Potential at Height h

$$\phi = -\frac{GM}{r} \qquad (\because r = R_e + h)$$

$$= -\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{4.24 \times 10^{7}}$$

$$\boxed{\phi = -9.43 \times 10^{6} \text{ J kg}^{-1}}$$

(128)



→ Gravitational field at P due to sphere A,

$$\vec{E}_1 = \frac{GM}{r^2} = \frac{G\,(100)}{(0.5)^2} \qquad \text{(Along PA)}$$

→ Gravitational field at P due to sphere B,

$$\vec{E}_2 = \frac{GM}{r^2} = \frac{G\,(100)}{(0.5)^2} \qquad \text{(Along PB)}$$

→ Both fields are equal and opposite so resultant gravitational field at P is Zero. Hence at that point gravitational force on particle having mass m is F = Im = 0

→ Total potential at point P

$$\phi = \phi_1 + \phi_2$$

$$= -\frac{GM}{r} - \frac{GM}{r} = -\frac{2\,GM}{r}$$

$$= -\frac{2 \times 6.67 \times 10^{-11} \times 100}{0.5}$$

$$\boxed{\phi = -2.668 \times 10^{-8} \text{ J kg}^{-1}}$$

→ By slightly shifting a particle of mass m from point P to left (or right) side, then due to increasing force in that direction. It will moves continuosly left side, (Right side). It will not return to P point, so particle of mass m can be consider as in unstable equilibrium.

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8 Escape Energy and Escape Speed :

→ If we throw a stone upwards with our hand, it goes to a certain height and then falls back towards the Earth. If we throw it with larger and larger initial speed we can send it to greater and greater heights. From this a natural question may arise : can we throw the stone with such an initial speed that it does not return back to Earth? It means, it goes to infinite distance from Earth forever and then there is no attraction on it by Earth. To get the answer let us consider its energy.

- Total energy -ve ⇒ Object Bound
- Total energy zero ⇒ (free) (steady)
- Total energy +ve ⇒ free (possess energy) In motion ($1/2\ mv^2$)

8.1 Formula

→ The gravitational potential energy of a body of mass m lying on the Earth's surface is $= \frac{-GM_e m}{R_e}$ and its kinetic energy is zero.

So its total enegy is $= \frac{-GM_e m}{R_e}$

→ If we supply energy $\frac{+GM_e m}{R_e}$ to this body in the form of kinetic energy, then it can go upto a point where its total energy becomes

$$\frac{+GM_e m}{R_e} + \left(\frac{-GM_e m}{R_e}\right) = 0.$$

→ It means, it will go to infinite distance from the Earth and there its potential energy is zero and kinetic energy is also zero.

(✦) If we give kinetic energy more than ($GM_e m$ / R_e) to the body then at infinite distance its potential energy becomes zero but it has still certain kinetic energy remaining with it.

(★) But depends on the mass and radius of the other body from the binding of which it has to escape.

→ In this condition the body ecapes from the binding with the Earth forever and does not return back. (✦)

→ **"The minimum energy to be supplied to the body to make it free from Earth's gravitational field (in other words from binding with the Earth) is called the ecape energy of that body." It is often called binding energy of the body.**

→ Thus, the escape energy of the body of mass m lying on the surface of the Earth

$$\text{escape energy} = \frac{GM_e m}{R_e} \quad \text{——(1)}$$

→ The minimum speed to be given to the body to give the kinetic energy equal to its escape energy is called the escape speed (v_e) which is often called the escape velocity also.

$$\therefore \frac{1}{2} m v_e^2 = \frac{GM_e m}{R_e}$$

$$\therefore \text{Escape speed } v_e = \sqrt{\frac{2GM_e}{R_e}} = \sqrt{\frac{2GM_e R_e}{R_e^2}} \quad \text{——(2)}$$

OR

$$v_e = \sqrt{2gR_e} \quad \text{——(3)}$$

OR $v_e = \sqrt{(8/3)\pi G \rho R_e^2}$ **(For surface only)**

Put $g = GM_e / R_e^2$

8.2 Understanding

- From eqn. (2) it is clear that the escape speed (v_e) of the body does not depends on its own mass.(★) does not depends on direction of projection.
- By putting the values of G , M_e and R_e in equation (2), we get $\boldsymbol{v_e}$ **= 11.2 km/s.**

→ If the initial speed of the body is equal to or greater than its escape speed (v_e), it will escape from the gravitational field of Earth forever. If the launching pad is at height h from surface of earth. $v_e = \sqrt{2GM_e / (R_e+h)}$ which is less than 11.2 km/s.

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8.3 Moon

→ The speed required for the body lying on the surface of moon, to make it free from the moon's gravitation is v_e' then

$$v_e' = \sqrt{\frac{2GM_m}{R_m}}$$

- where M_m = mass of the moon,
- R_m = radius of the moon.
- In that case v_e' = 2.3 km/s which is nearly (1/5) times the escape speed at the Earth's surface.
- Moon has no atmosphere because of this reason. If the gas molecules are formed on its surface then at the temperature prevailing there, those molecules have speeds greater than the above mentioned value. Hence, they escape the gravitational field of the moon forever.

8.4 Black Hole

From, $v_e = \sqrt{(8/3)\pi G\rho R^2}$

→ If the density (▲) of a body is so high that the escape speed (v_e) at its surface is ≥ velocity of light C, then nothing will be able to escape from its surface forever (not even light!) Such a body is called **black hole.**

→ We have to remember that no meterial particle can have velocity greater than or equal to the velocity of light $c = 3 \times 10^8$m/s

(▲) If density is more then mass is more so v is more from mass = volume × density

➥ **Note for escape speed :-**

- If we throw a body of mass m from surface of earth with initial speed u, then its total initial energy

$$E_i = \frac{1}{2}mu^2 + \left(-\frac{GM_e m}{R_e}\right)$$

- If its final velocity at height h from the surface of the earth is v, then final energy is

$$E_f = \frac{1}{2}mv^2 + \left(-\frac{GM_e m}{R_e + h}\right)$$

- According to law of conservation of M.E.

$$\frac{1}{2}mv^2 - \frac{GM_e m}{R_e + h} = \frac{1}{2}mu^2 - \frac{GM_e m}{R_e}$$

$$\therefore \frac{1}{2}mv^2 = \left(\frac{1}{2}mu^2 - \frac{GM_e m}{R_e}\right) + \frac{GM_e m}{R_e + h} \quad \ldots\ldots(i)$$

→ Total energy of system of any particle and earth is negative, a particle is binding with earth. If same positive total energy is given to particle, it will never come back to earth. equation (i)

→ According to

$$\frac{1}{2}mu^2 - \frac{GM_e m}{R_e} = 0 \ \& \ \frac{1}{2}mv^2 = \frac{GM_e m}{R_e + h}$$

→ So h is increasing and v is decreasing & at the end infinite height (h = infinite) $v = 0$

→ A particle is projected against gravitational field will never come back to earth till the velocity become zero.

If $\frac{1}{2}mu^2 - \frac{GM_e m}{R_e} \geq 0$,

- Then ½mv^2 never (at any height) becomes zero and body will never come back to earth so, to escape body, from earth forever, minimum initial velocity should be

$u \geq \sqrt{\frac{2GM_e}{R_e}}$, the minimum value of u is escape speed, $\therefore v_e = \sqrt{\frac{2GM_e}{R_e}}$

→ Here, we are neglected effect of other planets and starts. If $v < v_e$ a particle is reached other planet or space satellite due to its attraction force.

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Solved Problems

(130) Does the escape speed of a body from the earth depend on (a) the mass of the body, (b) the location from where it is projected, (c) the direction of projection, (d) the height of the location from where the body is launched ?

Solution :

→ (A) No, (B) Yes, (C) No, (D) Yes

→ v_e does not depend on mass of body which is projected and also not depend on direction of projection. It depend on place from which body launched and potential energy = $-GM_em/(R_e+h)$ it depends on location from which body is launched.

(131) The Earth retains its atmosphere because ....

(A) the value of the escape energy is less than the average kinetic energy of atmospheric molecules.

(B) the value of the escape energy is more than the average kinetic energy of atmospheric molecules.

(C) the Earth rotates

(D) the Earth is spherical

(132)The escape velocity on the surface of a planet is v_e. What would be the escape velocity on the planet having the same radius but mass 4 times that of it.

(A) $2v_e$ (B) $4v_e$ (C) v_e (D) $v_e/2$

Solution :

→ Escape velocity on planet $v_e = \sqrt{\frac{2GM}{R}}$

→ Escape velocity on big planet $v'_e = \sqrt{\frac{2G(4M)}{R}}$

$\therefore v'_e = 2\sqrt{\frac{2GM}{R}}$ $\therefore v'_e = 2v_e$

(133)If the densities of two planet are ρ_1 and ρ_2 and radii are R_1 and R_2 respectively, the ratio of the escape speed of the particle on their surface is v_1/v_2 = ...

(A) $\frac{\rho_1 R_1}{\rho_2 R_2}$ (B) $\sqrt{\frac{\rho_1 R_1}{\rho_2 R_2}}$

(C) $\sqrt{\frac{\rho_1 R_1^2}{\rho_2 R_2^2}}$ (D) $\frac{\rho_1^2 R_1^2}{\rho_2^2 R_2^2}$

Solution :

→ Escape speed of a planet having radius R_1

$$v_1 = \sqrt{\frac{2GM}{R_1}}$$

$$= \sqrt{\frac{2G \times \frac{4}{3}\pi R_1^3 \rho_1}{R_1}} = \sqrt{\frac{8\pi G \rho_1 R_1^2}{3}}$$

→ Escape speed of a planet having radius R_2

$$v_2 = \sqrt{\frac{8\pi G \rho_2 R_2^2}{3}}$$

$$\therefore \frac{v_1}{v_2} = \sqrt{\frac{\rho_1 R_1^2}{\rho_1 R_2^2}}$$

(134) Choose any one of the following four responses :

(A) If both Assertion and Reason are true and reason is the correct explanation of the Assertion.

(B) If both Assertion and Reason are true but Reason is not a correct explanation of the Assertion.

(C) If Assertion is true but Reason is false

(D) If both Assertion and Reason are false.

***Ans* : (131) B (132) A (133) C**

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Assertion :

Moon has no atmosphere.

Reason :

If the gas-molecules are formed on the surface of moon, the average kinetic energy of the molecule at the temperature prevailing there, is just equal to their potential energy.

(A) A (B) B (C) C (D) D

(135)The kinetic energy needed to project a body of mass m from the earth surface (radius R) to infinity is [AIEEE : 2002]

(A) *mg*R / 2 (B) 2*mg*R

(C) *mg*R (D) *mg*R / 4

***Solution* :**

→ Required kinetic energy for the body on the earth, $= \frac{GMm}{R}$

$$= \frac{GMm}{R^2}(R)$$

$$= mgR$$

(136) What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of 2R? [AIEEE : 2013]

(A) 5GmM / 6R (B) 2GmM / 3R

(C) GmM / 2R (D) GmM / 3R

***Solution* :**

→ Initial mechanical energy of satellite

$$E_i = K_i + U_i$$

$$= 0 + \left(-\frac{GM_m}{r_i}\right)$$

$$E_i = \frac{-GMm}{R}$$

→ Final mechanical energy of satellite

$$E_f = \frac{-GMm}{2r_f}$$

$$= \frac{1}{2}\left(\frac{-GMm}{3R}\right)$$

$$E_f = -\frac{GMm}{6R}$$

→ Minimum energy required to launch satellite

$$= E_f - E_i$$

$$= \frac{GMm}{R}\left(1 - \frac{1}{6}\right)$$

$$= \frac{5}{6}\frac{GMm}{R}$$

(137) Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}$ g. where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $\frac{2}{3}$ times that of the earth. If the escape speed on the surface of the earth is taken to be 11 km/s, the escape speed on the surface of the planet in km/s will be ....

[IIT : 2010]

(A) 1 (B) 2

(C) 3 (D) 4

***Solution* :**

→ $v = \sqrt{2gR}$

$$\frac{v_P}{v} = \sqrt{\frac{g_P}{g} \times \frac{R_P}{R}} \quad ... ...(1)$$

***Ans* : (134) C (135) C (136) A (137) C**

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$$\frac{g_P}{g_e} = \frac{\sqrt{6}}{11} \text{ given} \quad \ldots \ldots \ldots (2)$$

→ Use $g = \frac{4}{3} \pi G \rho R$

& get $\frac{R_P}{R}$ ... ...(3)

From (1), (2), (3) get answer of v_p .

(138) A satellite of 200 kg revolves around the Earth at a height of 1000 km from the surface of the Earth. Calculate its (1) escape energy (2) escape speed of this satellite.
Take G = 6.67 × 10^{-11} Nm^2/kg^2, raidus of the Earth = 6400 km and mass of the Earth = 6 × 10^{24} kg.

Solution :

→ m = 200 kg

h = 1000 km

r = 7400 km = 74 × 10^5 m

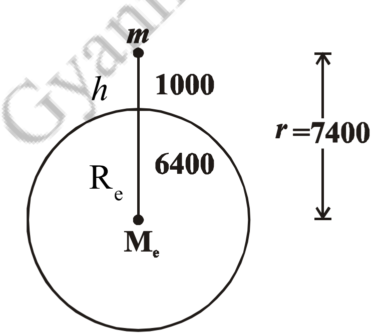
M_e = 6 × 10^{24} kg

R_e = 6400 km = 64 × 10^5 m(★)

G = 6.67 × 10^{-11} MKS

escape energy = ?

escape speed = ?



(★) (R_e + h)

★ **Escape energy :**

- centripetal force = gravitational force

$$\frac{mv^2}{r} = \frac{GM_e m}{r^2}$$

$$\tfrac{1}{2} mv^2 = \frac{GM_e m}{2r} \quad \boxed{(K)}$$

- potential energy $= -\frac{GM_e m}{r}$ $\boxed{(-2K)}$
- total energy,

= kinetic energy + potential energy.

$$E = \frac{GM_e m}{2r} - \frac{GM_e m}{r}$$

$$\boxed{E = -\frac{GM_e m}{2r}} \quad \boxed{(-K)}$$

$$E = -\frac{(6.67\times10^{-11})(6\times10^{24})\ (200)}{2\ (74\times10^5)}$$

$= -5.4 \times 10^9$ J

∴ $\boxed{\text{binding energy } B = 5.4 \times 10^9 \text{ J}}$

★ ***Escape Speed :***

$$v_e = \sqrt{\frac{2GM_e}{R_e+h}} = \frac{2(5.4 \times 10^9)}{200}$$

$$\boxed{v_e = 10.4\ km/s}$$

(139) A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expended to rocket the satellite out of the earth's gravitational influence ? Mass of the satellite = 200 kg; mass of the earth = 6.0 × 10^{24} kg ; radius of the earth = 6.4 × 10^6 m ; G = 6.67 × 10^{-11} Nm^2kg^{-2}.

Solution :

→ Total energy of the satellite in the orbit is,

$$E = \frac{-GMm}{2(R+h)}$$

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$$= -\frac{(6.67 \times 10^{-11})(6\times10^{24})(200)}{2(6.4 \times 10^6 + 400 \times 10^3)}$$

$= -5.89 \times 10^9$ J

→ Energy expended to rocket the satellite out of the earth's gravitational field
$= +5.89 \times 10^9$ J

(140) For an object lying on the surface of the Earth the escape speed is 11.2km/s. If an object on the Earth is thrown away with a speed three times this value, find its speed after it has escaped.

Solution :

→ From the gravitational field of the Earth

→ The initial speed of the object = $v = 3v_e$, where v_e = escape speed = 11.2 km/s.

→ Suppose the speed of this object after it escaped from the Earth's gravitational field (that is at infinite distance) = v'.

→ According to law of conservation of mechanical energy,

$$\begin{bmatrix}\text{Kinetic energy +}\\ \text{potential energy}\\ \text{at the Earth}\\ \text{surface}\end{bmatrix} = \begin{bmatrix}\text{Kinetic energy +}\\ \text{potential energy}\\ \text{at infinite}\\ \text{distance}\end{bmatrix}$$

$$\therefore \ \tfrac{1}{2} mv^2 + \left[\frac{-GM_e m}{R_e}\right] = [\tfrac{1}{2} mv'^2 + 0] \quad \text{—(1)}$$

(∵ at infinite distance potential en. = 0)

$$\text{But } v_e = \sqrt{\frac{2GM_e}{R_e}}$$

$$\therefore \frac{GM_e}{R_e} = \frac{v_e^2}{2}$$

→ Putting this value in equation (1) and writing $v = 3v_e$ (given), we get,

$$\frac{1}{2} m(9v_e^2) + \left[\frac{-v_e^2 m}{2}\right] = \frac{1}{2} mv'^2$$

$$\therefore \ 9v_e^2 - v_e^2 = v'^2 \ \Rightarrow \ \boxed{v' = v_e\sqrt{n^2-1} \text{ where n = 3}}$$

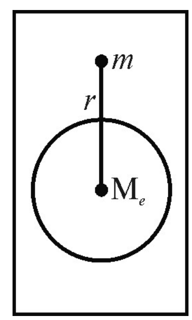
$$\therefore \ v' = \sqrt{8}\, v_e = (\sqrt{8})(11.2)$$

v' = 31.63 km/s

> **Note :** In this que. if an object is lying at a height equals to radius of the earth from the surface of the earth. Then find its speed after it has escaped. (19.4 km/s)

(141) An object is allowed to fall freely towards the Earth from a distance r (>R_e) from the center of the Earth. Find the speed of the object when it strikes the surface of the Earth.

Solution :



→ Allowing the body to fall freely from distance r > R_e, from the centre of Earth, its initial velocity is zero.
∴ Its kinetic energy = 0.

→ Its potential energy = $\frac{-GM_e m}{r}$
where m = mass of body.

→ When it strikes the surface of the Earth, if its velocity is v and the kinetic energy = ½ mv^2,
its potential energy here = $\frac{-GM_e m}{R_e}$

$$\begin{bmatrix}\text{Kinetic energy +}\\ \text{potential energy}\\ \text{at distance r from}\\ \text{Earth's surface}\end{bmatrix} = \begin{bmatrix}\text{Kinetic energy +}\\ \text{potential energy}\\ \text{at Earth's surface}\end{bmatrix}$$

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$$\therefore \left\{0+\left(\frac{-GM_e m}{r}\right)\right\} = \left\{\frac{1}{2}mv^2+\left(\frac{-GM_e m}{R_e}\right)\right\}$$

$$\therefore \boxed{v^2 = 2GM_e\left[\frac{1}{R_e}-\frac{1}{r}\right]} \quad \text{——— (1)}$$

$$\therefore \boxed{v = \sqrt{2GM_e\left[\frac{1}{R_e}-\frac{1}{r}\right]}}$$

→ This gives the required speed v.

→ To obtain the answer in terms of g, we write [write if it is asked]

$$g = \frac{GM_e}{R_e^2} \quad \therefore GM_e = gR_e^2$$

$$\therefore \quad v^2 = 2gR_e^2\left[\frac{1}{R_e}-\frac{1}{r}\right] \quad \text{——— (2)}$$

$$\therefore \boxed{v = \left[2gR_e^2\left(\frac{1}{R_e}-\frac{1}{r}\right)\right]^{\frac{1}{2}}} \quad \text{——— (3)}$$

Note : if it falls freely from a very large distance ($r \to \infty$) from the Earth's surface then equations (1) and (2) will give $v = \sqrt{\frac{2GM_e}{R_e}} = \sqrt{2gR_e}$. This is the same as the formula for the escape speed.

(142) Show that the ratio of the linear speed of a satellite revolving round the Earth and remainig close to the surface of the Earth to the escape speed of an object lying on the Earth is equal to ($1/\sqrt{2}$).

Ans :

→ $\frac{mv^2}{R_e} = \frac{GM_e m}{R_e^2}$ (where v = orbital velocity)

$$\therefore \quad v = \sqrt{\frac{GM_e}{R_e}} \quad \text{——— (1)}$$

→ The escape velocity on the surface of Earth for a body (at rest) is given by

$$v_e = \sqrt{\frac{2GM_e}{R_e}} \quad \text{——— (2)}$$

→ From eqn. (1) and (2)

$$\frac{v}{v_e} = \frac{1}{\sqrt{2}}$$

(143) Potential energy of a satellite having mass m and rotating at a height of 6.4×10^6 m from the earth centre is : [AIIMS : 2000]

(A) – 0.2 mg R_e (B) – 2 mg R_e

(C) – 0.5 mg R_e (D) – mg R_e

Solution :

→ For total energy (Ans. : C)

(144) A satellite is in an orbit around the earth. If its kinetic energy is doubled, then [AIIMS-2014]

(A) it will maintain its path

(B) it will fall on the earth

(C) it will rotate with a great speed

(D) it will escape out of earth's gravitational field

Solution :

→ Orbital speed, $v_0 = \sqrt{gr} = \sqrt{GM_e/r}$ and escape speed $v_e = \sqrt{2gr}$

→ Now kinetic energy is doubled then speed becomes $\sqrt{2}$ times

→ So, particle will escape out of earth's gravitational field.

Ans **: (143) D (144) D**

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(145) A satellite is revolving in a circular orbit at a height 'h' from earth's surface (radius of earth R : h << R). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to : (Neglect the effect of atmosphere) [JEE-2016]

(A) $\sqrt{2gR}$ **(B)** $\sqrt{gR}$

(C) $\sqrt{gR/2}$ **(D)** $\sqrt{gR}(\sqrt{2}-1)$

Solution :

→ **Orbital speed**

$$v_o = \sqrt{\frac{GM}{R}} = \sqrt{\frac{GMR}{R^2}} = \sqrt{gR}$$

→ Escape speed $v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2GMR}{R^2}} = \sqrt{2gR}$

→ Increase in speed $= \sqrt{2gR} - \sqrt{gR}$

$$= \sqrt{gR}\left(\sqrt{2}-1\right)$$

(146) A satellite revolves around the earth remaining quite close to the surface of the Earth. How much additional velocity should be given to it to make it free from the binding of the Earth ? Near the Earth's surface take $g = 10\ ms^{-2}$. radius of the earth $R_e = 6400\ km$.

Solution :

→ Orbital speed, $v_0 = \sqrt{gr}$ and escape speed $v_e = \sqrt{2gr}$

Extra speed is given to satelite,

$= \sqrt{2gr} - \sqrt{gr} \quad = (\sqrt{2}-1)\sqrt{gr}$

$= 0.414\sqrt{(10)(64\times10^5)}$

$= 331.2$ m/s

***Ans* : (145) C (147) D (148) D**

(147) The addiational kinetic energy to be provided to a satellire of mass m revolving around a planet of mass M to transfer it from a circular orbit of radius R_1 to another of radius R_2 ($R_2 > R_1$) is : [AIIMS-2016]

(A) $GmM\left(\frac{1}{R_1^2} - \frac{1}{R_2^2}\right)$

(B) $GmM\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

(C) $2GmM\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

(D) $\frac{1}{2}GmM\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Solution :

$$\rightarrow \quad Ef - Ei = \frac{-GMm}{2r_f} - \left(\frac{-GMm}{2r_i}\right)$$

$$= \frac{GMm}{2}\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

(148) A satellite of mass m is in circular orbit of radius $3R_E$ about earth (mass of earth M_E , radius of earth R_E). How much additional energy is required to transfer the satellite to an orbit of radius $9R_E$? [NEET - 2017]

(A) $\frac{GM_Em}{3R_E}$ **(B)** $\frac{GM_Em}{18R_E}$

(C) $\frac{3GM_Em}{2R_E}$ **(D)** $\frac{GM_Em}{9R_E}$

Solution :

$$\rightarrow \quad \Delta U = \frac{GMm}{2}\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

can be directly used

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→ Total Energy of Satellite $E = -\frac{GM_e m}{2r}$

$r = 3R_E \rightarrow E_1 = -\frac{GM_e m}{6R_E}$

$r = gR_E \rightarrow E_2 = -\frac{GM_e m}{18R_E}$

→ Required Additional Energy

$$\Delta E = -\frac{GM_E m}{18R_E} - \left(-\frac{GM_E m}{6R_E}\right)$$

$$\therefore \Delta E = -\frac{2GM_E m}{18R_E}$$

$$\boxed{\Delta E = \frac{GM_E m}{9R_E}}$$

(149) A body attains a height equal to the radius of the earth. The velocity of the body with which it was projected is ....

[NEET-2001]

(A) $\sqrt{\frac{GM}{R}}$ **(B)** $\sqrt{\frac{2GM}{R}}$

(C) $\sqrt{\frac{5}{4}\frac{GM}{R}}$ **(D)** $\sqrt{\frac{3GM}{R}}$

Solution :

→ Total energy at surface of the earth = Total energy at maximum height.

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = \frac{1}{2}m(0)^2 - \frac{GMm}{R+h}$$

→ At maximum height it has only potential energy.

$$\therefore \frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{R+R}$$

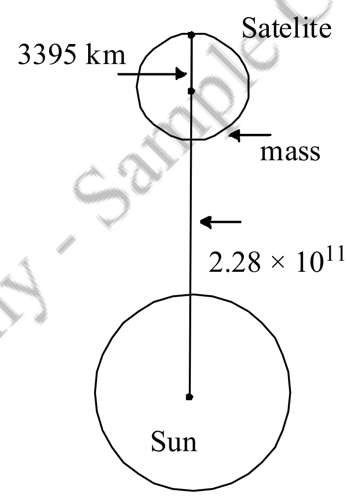
$$v^2 = \frac{2GM}{R} - \frac{2GM}{2R} \qquad v = \sqrt{\frac{GM}{R}}$$

***Ans* : (149) A (151) C**

(150) A spaceship is stationed on Mars. How much energy must be expended on the spaceship to launch it out of the solar system ? Mass of the space ship = 1000kg; mass of the sun = 2 × 10^30^kg; mass of mars = 6.4 × 10^23^ kg; radius of mars = 3395 km; radius of the orbit of mars = 2.28 × 10^8^ km ; G = 6.67 × 10^-11^ N m^2^ kg^-2^.

***Solution* :**

→



® $r = 2.28 \times 10^{11} + 3395 \times 10^3$

→ $E = -\frac{GMm}{2r}$

→ $E_e = +\frac{GMm}{2r}$

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(151) Escape velocity from earth is 11.2 km/s. Another planet of same mass has radius 1/4 times of the earth. What is the escape velocity from another planet ? [NEET : 2000]

(A) 11.2 km/sec (B) 44.8 km/sec

(C) 22.4 km/sec (D) 5.6 km/sec

Solution :

→ From,

$$v_e = \sqrt{\frac{2GM}{R}}$$

→ If M is constant

$$v_e \propto \sqrt{\frac{1}{R}}$$

$\therefore v_e$ (planet) = 2 v_e (earth) = 22.4 km/h

Notes

Notes

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Current Topic Practice

(152) The escape velocity of a sphere of mass m is given by (G = universal gravitational constant, M_e = mass of the earth and R_e = radius of the earth)

[NEET-2006]

(A) $\sqrt{\frac{GM_e}{R_e}}$ (B) $\sqrt{\frac{2GM_e}{R_e}}$

(C) $\sqrt{\frac{2Gm}{R_e}}$ (D) $\frac{GM_e}{R_e^2}$

(153) The escape velocity of a body depends upon mass as [AIEEE-2002]

(A) m^0 (B) m^1 (C) m^2 (D) m^3

(154) The velocity with which a projectile must be fired so that it escapes earth's gravitation does not depend on :

[AIIMS-2002]

(A) mass of the earth

(B) mass of the projectile

(C) radius of the projectile's orbit

(D) gravitational constant

(155) The escape velocity of a body on the surface of the earth is 11.2 km/s. If the earth's mass increases to twice its present value and the radius of the earth becomes half, the escape velocity would become...

[NEET-1997]

(A) 44.8 *km/sec* (B) 22.4 *km/sec*

(C) 11.2 *km/sec* (D) 5.6 *km/sec*

(156) The earth is assumed to be a sphere of radius R. A platform is arranged at a height R from the surface of the earth. The escape velocity of a body from this platform is fv_e where v_e is its escape velocity from the surface of the earth. The value of f is ..... [NEET-2006]

(A) $\sqrt{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$

(157) The ratio of the radii of two planets r_1 and r_2 is k. The ratio of acceleration due to gravity on them is r. Then the ratio of the escape velocities from them, will be:

[AIIMS-1997]

(A) $\sqrt{r/k}$ (B) $\sqrt{k/r}$

(C) kr (D) $\sqrt{kr}$

(158) The escape velocity from the earth is 11.2 km/sec. The escape velocity from a planet having twice the radius and the same mean density as the earth, is :

[AIIMS-2001]

(A) 11.2 km/sec (B) 22.4 km/sec

(C) 15.00 km/sec (D) 5.8 km/sec

(159) The ratio of escape velocity at earth (v_e) to the escape velocity at a planet (v_p) whose radius and mean density are twice as that of earth is :

[NEET : PHASE : 1 (2016)]

(A) $1 : 2\sqrt{2}$ (B) 1 : 4

(C) $1 : \sqrt{2}$ (D) 1 : 2

(160) The escape velocity from the surface of the earth is v_e. The escape velocity from the surface of a planet whose mass and radius are three times those of the earth, will be ..... [NEET-1995]

(A) v_e (B) $3v_e$ (C) $9v_e$ (D) $1/3v_e$

***Ans* : (152) B (153) A (154) B (155) B (156) B (157) D (158) B (159) A (160) A**

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(161)For a satellite escape velocity 11 km/s. If the satellite is launched at an anlge of 60^0 with the vertical, then escape velocity will be [NEET-1989]

(A) 11 *km/sec* (B) 11 $\sqrt{3}$ *km/sec*

(C) $\frac{11}{\sqrt{3}}$ *km/sec* (D) 33 *km/sec*

(162) The escape velocity of an object lying on the surface of Earth is v_e. If an object on the earth is thrown away with a speed n time this value then prove that its speed after it has escaped from the graviational field of the Earth is

$v' = v_e \sqrt{(n^2 - 1)}$

(163) For an object lying on the surface of the Earth, the escape speed is 11.2 km/s. If the Jupiter becomes 318 times heavier than that of Earth and the radius of Jupiter becomes 11.2 time that of Earth. Then find the escape speed of an object from the Jupiter.

(164) For an object lying on the surface of the Earth, the escape speed is 11.2 km/s. If an object on the Earth is thrown away with a speed two times this value, find its speed after it has escaped from the gravitational field of the Earth.

(165) A space shuttle revolves in a circular orbit near the surface of the Earth. If it has escaped from the gravitational field of the Earth how much more velocity to be given to the space shuttle? Radius of the Earth = 6400 km. g = 9.8 m/s^2

(166) The escape velocity for a body projected vertically upwards from the surface of earth is 11 km/s. If the body is projected at an angle of 45^0 with the vertical, the escape velocity will be ....

[AIEEE-2003]

(A) 11 $\sqrt{2}$ *km/s*

(B) 22 *km/s*

(C) 11 *km/s*

(D) 11/ $\sqrt{2}$ *km/s*

(167) A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 km/s, the escape velocity from the surface of the planet would be

[AIEEE-2008]

(A) 1.1 *km/sec* (B) 11 *km/sec*

(C) 110 *km/sec* (D) 0.11 *km/sec*

(168) Knowing that the mass of the moon is (1 / 81) times that of earth and its radius is (1/4) the radius of earth. If the escape velocity at the surface of the earth is 11.2 km/sec, then the value of escape velocity at the surface of the moon is :

[AIIMS-2000]

(A) 2.5 km/sec (B) 0.14 km/sec

(C) 5 km/sec (D) 8 km/sec

(169) A rocket is fired vertically with a speed of 5 km s^{-1} from the earth's surface. How far from the earth does the rocket go before returning to the earth ? Mass of the earth = 6.0×10^{24} kg; mean radius of the earth = 6.4×10^6 m;

G = 6.67×10^{-11} $Nm^2 \, kg^{-2}$.

***Ans* : (161) A (166) C (167) C (168) A**

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(170) A satellite is moving a constant speed 'V' in a circular orbit about the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is .... [IIT-2011]

(A) $(1/2)\ mV^2$ (B) mV^2

(C) $(3/2)\ mV^2$ (D) $2\ mV^2$

(171) Two spherical planets P and Q have the same uniform density ρ, masses M_P and M_Q and surface areas A and 4A respectively. A spherical planet R also has uniform density ρ and its mass is $(M_P + M_Q)$. The escape velocities from the planets P, Q and R are V_P, V_Q and V_R, respectively. Then .... [IIT-2012]

(A) $V_Q > V_R > V_P$ (B) $V_R > V_Q > V_P$

(C) $V_R / V_P = 3$ (D) $V_P / V_Q = (1/2)$

Notes

Ans : (170) B (171) B,D

Hints & Solutions

153.

Escape Velocity $v_e = \sqrt{\frac{2GM_e}{R_e}}$

is independent of mass of object

155.

Escape Velocity on the earth's surface is given by

$$v_{es} = \sqrt{\frac{2GM_e}{R_e}} = 11.2 \text{ km/s}$$

$$M_e' = 2m_e$$

$$R_e' = \frac{R_e}{2}$$

$$\therefore \frac{v'_{es}}{v_{es}} = \sqrt{\frac{2M_e}{M_e} \times \frac{R_e}{R_e/2}}$$

$$= \sqrt{4}$$

$$= 2$$

$$\therefore v'_{es} = 2 v_{es}$$

$$= 2 \times 11.2$$

$$v'_{es} = 22.4 \text{ km/s}$$

156. Escape energy at a platform of height R from the earth's surface

$$\frac{1}{2} m (fv)^2 = \frac{GMm}{2R}$$

$$\therefore f\ v = \sqrt{\frac{2GM}{2R}}$$

$$v_e = \sqrt{\frac{2GM}{R}} \quad \text{(on earth's surface)} \text{ ——— (ii)}$$

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$$\frac{f\ v_e}{v_e} = \sqrt{\frac{2Gm}{2R}}\sqrt{\frac{R}{2Gm}}$$

$$\boxed{f = \frac{1}{\sqrt{2}}}$$

157.

→ $v_e = \sqrt{2gR}$

→ $\dfrac{(v_e)_1}{(v_e)_2} = \dfrac{\sqrt{2gR_1}}{\sqrt{2gR_2}}$

$= \sqrt{kr}$

159.

$$v_e = \sqrt{\frac{2GM}{R}}$$

$$= \sqrt{\frac{8}{3}\pi G R^2 \rho}$$

$$\frac{v_e}{v_p} = \sqrt{\left(\frac{R_e}{R_P}\right)^2 \left(\frac{\rho_e}{\rho_P}\right)}$$

$$\frac{v_e}{v_P} = \frac{1}{2\sqrt{2}}$$

160.

$M_p = 3M_e$

$R_P = 3R_e$

$$v_e = \sqrt{2g\,R_e} = \sqrt{\frac{2GM_e}{R_e}}$$

$$v_e = \sqrt{\frac{M_e}{R_e}}$$

$$\therefore \quad \frac{v_e}{v_p} = \sqrt{\frac{M_e\ R_p}{R_e\ M_P}}$$

$$= \sqrt{\frac{M_e\ (3R_e)}{R_e\ (3M_e)}}$$

$$\frac{v_e}{v_P} = \frac{1}{1}$$

$$\therefore\ v_e = v_P$$

161.

→ Escape velocity is independent from angle of projection.

162.

$$\frac{1}{2}mn^2\frac{v^2}{v_e} - \frac{GM_e m}{R_e} = \frac{1}{2}mv'^2$$

$$m\left[\frac{1}{2}n^2 v_e^2 - \frac{GM_e}{R_e}\right] = \frac{1}{2}mv'^2$$

$$\frac{v'^2}{2} = \frac{1}{2}n^2 v_e^2 - \frac{GM_e}{R_e}$$

But $v_e = \sqrt{\dfrac{2GM_e}{R_e}}$

$$\therefore \frac{GM_e}{R_e} = \frac{v_e^2}{2}$$

$$\frac{v'^2}{2} = \left(n^2 - 1\right)v_e^2$$

$$\boxed{v' = v_e\sqrt{n^2 - 1}}$$

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163.

$$\frac{v_j^2}{v_e^2} = \frac{2GM_j / R_j}{2GM_e/R_e}$$

$$\frac{v_j^2}{v_e^2} = 28.39$$

$$v_j = \sqrt{28.39}\ v_e$$

$$= 59.68\ \text{km/s}$$

164.

$$\frac{1}{2}\ m\ (2v_e)^2 - \frac{GM_e m}{R_e} = \frac{1}{2} m v'^2$$

$$\frac{4v_e^{\ 2}}{2} - \frac{GM_e}{R_e} = \frac{1}{2}\ v'^2$$

$$\therefore\ v'^2 = 3v_e^{\ 2}$$

$$\therefore\ v' = \sqrt{3} v_e$$

$$v' = 19.39\ \text{km/s}$$

165.

$$E = K + U$$

$$= -\frac{GM_e m}{2R_e}$$

$$\therefore\ \frac{1}{2}\ m\ v^2 = +E$$

$$\frac{1}{2}\ m\ v^2 = \frac{GM_e m}{2R_e}$$

$$\therefore\ v^2 = gR_e$$

$$\therefore\ v = \sqrt{g\,R_e}$$

$v = 7920$ m/S

Extra velocity $= 11200 - 7920$

$= 3.3 \times 10^3$ m/s

166.

→ Escape velocity is independent from angle of projection.

167.

→ $M_P = 10M_e$

$$R_P = \frac{R_e}{10}$$

$v_e = 11$ km s^{-1}

$v_P = ?$

→ For earth : $v_e = \sqrt{\frac{2GM_e}{R_e}} = 11$ km/s

→ For planet : $v_P = \sqrt{\frac{2GM_P}{R_P}}$

$$= \sqrt{\frac{2G(10M_e)}{\frac{R_e}{10}}}$$

$$= 10\sqrt{\frac{2GM_e}{R_e}}$$

$v_P = 10\ v_e = 110$ km/s

168.

→ Escape velocity $v_e = \sqrt{\frac{2GM}{R}}$

→ $v'_e = \sqrt{\frac{2GM/81}{R/4}}$

$$= \frac{2}{9} v_e$$

$$= \frac{2}{9}(11.2)$$

$v'_e = 2.5$ km/s

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169.

$u = 5\ \text{kms}^{-1} = 5000\ \text{ms}^{-1}$

$M = 6 \times 10^{24}$ kg

$R = 6.4 \times 10^{6}$ m

$G = 6.67 \times 10^{-11}\ \text{Nm}^2\ \text{kg}^{-2}$

→ Suppose the rocket goes upto a height h before returning to the earth.

→ At this height velocity of rocket will become zero.

→ By the conservation of energy

Total energy of earth's surface

= Total energy at height h

$$\frac{1}{2}mu^2 - \frac{GMm}{R} = 0 - \frac{GMm}{R+h}$$

[In $\frac{1}{2}mv^2$ final velocity is zero. So object (body) return to earth.]

$$\frac{1}{2}u^2 = \frac{GM}{R} - \frac{GM}{R+h}$$

$$\frac{1}{2}u^2 = \frac{gR^2}{R} - \frac{gR^2}{R+h}$$

$$\frac{1}{2}u^2 = gR\left(1 - \frac{R}{R+h}\right)$$

$$\frac{1}{2}u^2 = gR\left(\frac{h}{R+h}\right)$$

$$h = \frac{Ru^2}{2gR - u^2}$$

$$= \frac{(6.4\times10^6)(5\times10^3)^2}{2(9.8)(6.4\times10^6) - (5\times10^3)^2}$$

$h = 1.6 \times 10^6$ m

170.

V is orbital velocity & v_e is escape velocity then,

$$v_e = \sqrt{2}\ V$$

Kinetic energy at time of projection is

$$K = \frac{1}{2}m\,v_e^2$$

$$= \frac{1}{2}m\left(\sqrt{2}\ V\right)^2$$

$$= m V^2$$

171.

Suppose mass of planet P is m. Radius of planet P is r.

$$\therefore\ m = \rho v = \rho\left(\frac{4}{3}\pi r^3\right)$$

$$= \rho\left(\frac{4}{3}\pi\left(\frac{A}{4\pi}\right)^{3/2}\right)$$

Mass of Q $= \rho\left(\left(\frac{4}{3}\right)\pi\left(\frac{4A}{4\pi}\right)^{3/2}\right) = 8$ m

Mass of R = 9 m

Radius of Q is 2r and Radius of $R = 9^{1/3}r$

Mass of R = 9m
$\frac{4}{3}\pi r'^3\rho = 9\left(\frac{4}{3}\pi r^3\right)\rho$
$r' = 9^{1/3}r$

$$v_P = \sqrt{\frac{2GM_P}{R_P}}$$

$$v_Q = \sqrt{\frac{2GM_Q}{R_Q}}$$

$$v_R = \sqrt{\frac{2GM_R}{R_R}}$$

Use it and get the answer

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9 Satellites :

9.1 Satellite

→ **A body revolving around a planet is called its satellite.**

9.1.1 Types

→ Satellites can be classified into two categories :

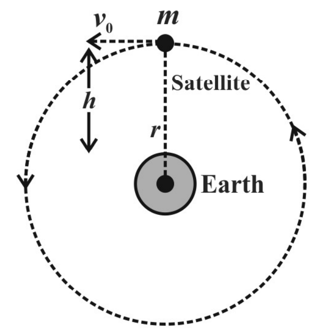
(1) **Natural Satellite**

- Moon is the natural satellite of the Earth.
- Moreover, Jupiter and other planets also have their moons (means satellites).
- The periodic time of our **moon's revolution around the Earth** is **27.3 days** and the periodic time of rotation of moon about its own axis is also nearly the same.

(2) **Artificial Satellite**

- The first artificial satellite made by the mankind was " **Sputnic** " put into orbit around the Earth by **Russian scientsts** in 1957.
- Our Indian scientists have also successfully launched **'Aryabhatta'** and **'INSAT'** series of satellites.
- Presently hundreds of satellites launched by many countries of the world around the Earth. They are used for scientific, engineering, communication, whether forecast, spying and military purposes.
- In the present article we shall study the dynamics of the satellite and geo-stationary (or geo-synchronous) as well as polar satellites.

9.2 Total energy of Satellite



Orbital motion of a satellite

→ Suppose a satellite of mass m is launched at distance r from the center of the Earth and its speed in the circular orbit is v_0.It is also called the **orbital speed or the orbital velocity.**

- Here $r = R_e + h$ where R_e = radius of the Earth, h = height of the satellite from the Earth's surface.

→ The necessary centripetal force (mv_0^2/r) for this circular motion of the satellite is provided by the Earth's gravitational force on it.

$$\therefore \boxed{\frac{mv_0^2}{r} = \frac{GM_e m}{r^2}} \quad \text{—— (1)}$$

∴ The orbital speed of the satellite is

$$\boxed{v_0 = \sqrt{\frac{GM_e}{r}}} \quad \text{—— (2)}$$

→ From equation (1), the kinetic energy of the satellite is, $\boxed{K = \frac{1}{2} mv_0^2 = \frac{GM_e m}{2r}}$ —— (3)

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→ The potential energy of this satellite (actually of the system of Earth + satellite)

$$U = \frac{-GM_e m}{r} \qquad \text{———— (4)}$$

∴ Total energy of the satellite is,

E = kinetic energy K + potential energy U

$$= \frac{GM_e m}{2r} - \frac{GM_e m}{r}$$

$$E = \frac{-GM_e m}{2r} \qquad \text{———— (5)}$$

- This total energy is negative, which indicates that this satellite is in the bound state.
- You will be able to see from equations (3), (4) and (5) that if the kinetic energy of the satellite is x, its potential energy is $-2x$ and the total energy is $-x$. Hence its binding energy also equal to its escape energy is x.

9.3 Time period (T) of the satellite :

→ The time taken by the satellite to complete one revolution around Earth is called its time-period or the periodic time or the period (T) of revolution. During this time the distance travelled by it is equal to the circumference (= $2\pi r$) of the circular path.

∴ The orbital speed $v_0 = \frac{2\pi r}{T}$ ———— (6)

∴ From the equation $\frac{mv_0^2}{r} = \frac{GM_e m}{r^2}$

$$\frac{m}{r}\left(\frac{4\pi^2 r^2}{T^2}\right) = \frac{GM_e m}{r^2}$$

$$\therefore T^2 = \left(\frac{4\pi^2}{GM_e}\right) r^3 \qquad \text{———— (7)}$$

→ Since all quantities in the bracket are constant we can say that

$T^2 \propto r^3$

→ Thus, **"the square of the orbital time-period of the satellite is directly proportional to the cube of the orbital radius."**

- This is Kepler's third law with reference to the circular orbit of the satellite.

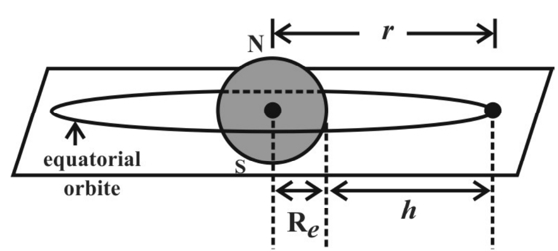
From equation (7),

$$T = \left(\frac{4\pi^2 r^3}{GM_e}\right)^{\frac{1}{2}} \qquad \text{———— (9)}$$

9.4 Geo-stationary satellite :

→ The Earth's satellite having orbital periodic time of 24 hours (equal to the periodic time of rotation of the Earth about its own axis), is called geo-stationary satellite (or geo-sychronous satellite), because it appears always stationary as viewed from the Earth.

- Such a geo-stationary revolve around the Earth in the equatorial plane in east-west direction. See following figure.



geo stationary satellite

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→ For geo-stationary satellite by putting
$G = 6.67 \times 10^{-11}\ Nm^2\ kg^{-2}$,
$m_e = 5.98 \times 10^{24}$ kg and
$T = 24 \times 3600$ s, in

$$T = \left(\frac{4\pi^2 r^3}{GM_e}\right)^{\frac{1}{2}}$$

we get r = 42260 km.

→ Hence the height of this geo-stationary satellite from the Earth's surface is

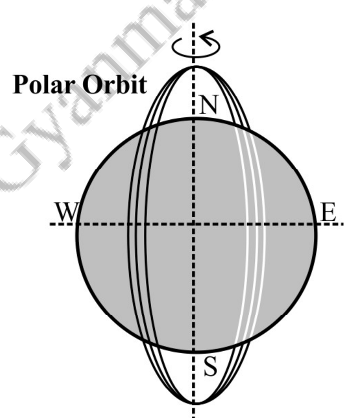
$$h = r - R_e = 42260 - 6400 = 35860 \text{ km.}$$

- A satellite cannot remain geo-stationary for any other height except this one.

→ **Uses**

- These satellites are used in telecommunication.
- Moreover they are also used in **Global Positioning System (GPS)** in which a person gets information about various ways and the shortest route to go from his present position to his destination,alongwith the map displayed on the screen of the monitor.

9.5 Polar Satellite :



Polar Orbit

→ These satellites revolve around the Earth in north-south direction.

- Their heights from the suface of the Earth is nearly 800 km.

→ Since the Earth rotates in the east-west direction, these satellites (Their time-period is almost 100 min.) Can view every section (♠) of the Earth many times in a day. With the help of a camera kept inside this satellite it can see a thin strip of the Earth in every rotation.

→ In the next rotation it will see the region of the next strip.

→ Thus can see the entire Earth many times in a day.

→ They are useful in remote sensing, meteorology, environmental study, spying etc.

10. Weightlessness :

→ Weight of an object is the force with which the earth attracts it.

→ We are conscious of our own weight when we stand on a surface, since the surface exerts a force opposite to our weight to keep us at rest.

→ The same principle holds good when we measure the weight of an object by a spring balance hung from a fixed point e.g. the ceiling.

→ The object would fall down unless it is subject to a force opposite to gravity. This is exactly what the spring exerts on the object.

(♠) The cameras kept at this height can capture only very tiny portion like thin slit.

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→ This is because the spring is pulled down a little by the gravitational pull of the object and in turn the spring exerts a force on the object vertically upwards.

→ Now, imagine that the top end of the balance is no longer held fixed to the top ceiling of the room.

→ Both ends of the spring as well as the object move with identical acceleration g.

→ The spring is not stretched and does not exert any upward force on the object which is moving down with acceleration g due to gravity.

→ The reading recorded in the spring balance is zero since the spring is not stretched at all.

→ If the object were a human being, he or she will not feel his weight since there is no upward force on him.

→ Thus, when an object is in free fall, it is weightless and this phenomenon is usually called the phenomenon of weightlessness.

→ In a satellite around the earth, every part and parcel of the satellite has an acceleration towards the center of the earth which is exactly the value of earth's acceleration due to gravity at that position.

→ Thus in the satellite everything inside it is in a state of free fall.

→ This is just as if we were falling towards the earth from a height.

→ Thus, in a manned satellite, people inside experience no gravity.

→ Gravity for us defines the vertical direction and thus for them there are no horizontal or vertical directions, all directions are the same.

→ Pictures of astronauts floating in a satellite reflect this fact.

Notes

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Solved Problems

(172)Two satellites revolving around a planet in the same orbit have the ratio of their masses $(m_1/m_2) = (1/2)$. The ratio of their orbital velocities (v_1 / v_2) = ...

(A) 1 (B) $\frac{1}{2}$ (C) 2 (D) 4

Solution :

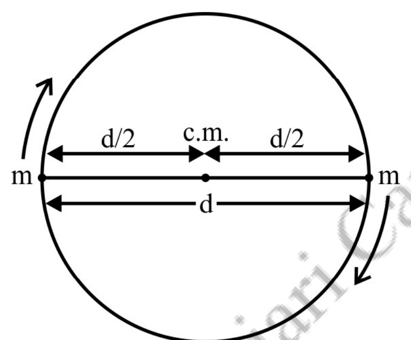
⇒ In orbital velocity $v_0 = \sqrt{GM_e / r}$ mass is not present, therefore the orbital velocity does not depend upon the mass.

(173) The distance between two bodies, each of mass m is d. If they perform circular motion around their centre of mass; what is their orbital velocity ?

(A) $\sqrt{Gm / d}$ (B) $\sqrt{Gm / 4d}$

(C) $\sqrt{Gm / 3d}$ (D) $\sqrt{Gm / 2d}$

***Solution* :**



→ Centripetal force = Gravitational force

$$\frac{mv^2}{\frac{d}{2}} = \frac{Gmm}{d^2}$$

$$\therefore 2v^2 = \frac{Gm}{d}$$

$$\therefore v^2 = \frac{Gm}{2d}$$

$$\therefore v = \sqrt{\frac{Gm}{2d}}$$

(174) The radii of circular orbits of two satellites A and B of the earth are 4R and R, respectively. If the speed of satellite A is 3*v*, then the speed of satellite A is 3*v*, then the speed of satellite B will be.... [NEET-2010]

(A) 3 *v* / 4 (B) 6 *v*

(C) 12 *v* (D) 3 *v* / 2

***Solution* :**

→ Orbital speed of satellite

$$\therefore \frac{v_A}{v_B} = \sqrt{\frac{r_B}{r_A}}$$

$$= \sqrt{\frac{R}{4R}}$$

$$= \frac{1}{2}$$

$$\frac{v_A}{v_B} = \frac{3v}{v_B} = \frac{1}{2}$$

$$v_B = 6v$$

(175)A satellite revolves in a circular orbit around the earth. If the gravitational force on it by the Earth suddenly disappears; then

(A) it will move with the same speed in the tangential direction to its orbit at that instant

(B) it will move with the same speed on its orbit

(C) it will fall towards the Earth with an accelerated motion

(D) it will become stationary at the point

***Ans* : (172) A (173) D (174) B (175) A**

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(176) Consider different planets revolving in different circular orbits around the star of very large mass If the gravitational force between the planet and the star varies as $r^{-5/2}$, r = distance between them. How does the square of the orbital period T depend on the distance r ?

Solution :

→ r = orbital radius of the planet around star

M = mass of the star

m = mass of the planet

→ required centripetal force = gravi. force

$$\frac{mv^2}{r} = \frac{GMm}{r^{5/2}}$$

$$\left(\frac{2\pi r}{T}\right)^2 = \frac{GMr}{r^{5/2}}$$

$$\frac{4\pi^2}{T^2} r^2 = \frac{GMr}{r^{5/2}}$$

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^{7/2}$$

$$\therefore \ T^2 \propto r^{7/2}$$ (✱)

(177) If the time period of a satellite in the orbit of radius r around a planet is T, then the time period of a satellite in the orbit of radius 4r is T' = ...

(A) 4T (B) 2T (C) 8T (D) 16T

Solution :

⇒ $T \propto r^{3/2}$

$$\therefore \frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} = \left(\frac{4r}{r}\right)^{3/2}$$

$$\therefore \frac{T_2}{T} = 8$$

$$\therefore T_2 = 8\ T$$

(✱) If $F \propto r^{-n}$ *then* $T^2 \propto r^{n+1}$

If $F \propto r^{-2}$, then $T^2 \propto r^3$

(178) The time period of a satellite of earth is 5 hours. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time period will become ..... hours.

[AIEEE-2003]

(A) 10 (B) 80 (C) 40 (D) 20

Solution :

→ According to the Kepler's third law,

→ $T^2 \propto r^3$

→ $$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

→ $$T_2^2 = \frac{r_2^3}{r_1^3} T_1^2$$

→ $T_2^2 = (4)^3\ (5)^2$

∴ $T_2 = 40$ h

(179) If the distance between the earth and the sun were half its present value, the number of days in a year would have been .... [IIT-1996]

(A) 64.5 (B) 129 (C) 182.5 (D) 730

(180) From the surface of the earth at height of 5R a geostationary satellite is moving. Where R is radius of the earth. At the height of 2R from the surface of the earth, for another satellite. The time period of satellite will be .... hours.

[NEET : 2012]

(A) 5 (B) 10 (C) $6\sqrt{2}$ (D) $\frac{6}{\sqrt{2}}$

Solution :

→ According to the Kepler's third law,

∴ $T^2 \propto r^3$

Ans **: (177) C (178) C (179) B (180) C**

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$$\therefore \quad \frac{T_2^2}{T_1^2} = \frac{r_2^3}{r_1^3} = \frac{(3R)^3}{(6R)^3}$$

$$\therefore \quad T_2^2 = \frac{T_1^2}{8} = \frac{24}{2\sqrt{2}}$$

$$\therefore \quad T_2 = 6\sqrt{2}\ h$$

(181) What is the nature of relation between the kinetic energy (E_k) and their orbital radius (r) of the satellites revolving around the Earth ?

(A) $E_k \propto r$ **(B) $E_k \propto (1/r)$**

(C) $E_k \propto r^2$ **(D) $E_k \propto (1/r^2)$**

Solution :

⇒ Centripetal force = Gravitation force

$$\frac{mv^2}{r} = \frac{GM_e m}{r^2} \quad \therefore \tfrac{1}{2} mv^2 = \frac{GM_e m}{2r}$$

$$\therefore \quad E_k = \left(\frac{GM_e m}{2}\right) \cdot \frac{1}{r}$$

$$\therefore \quad E_k \propto \frac{1}{r} \qquad \left(\frac{GM_e m}{2} \text{ is constant}\right)$$

(182) How does the kinetic energy of different satellites revolving around a planet depend on their periodic time T ?

(A) proportional to $T^{-2/3}$

(B) proportional to $T^{2/3}$

(C) proportional to $T^{3/2}$

(D) proportional to $T^{-3/2}$

Solution :

→ Kinetic Energy $K = \frac{GMm}{2r}$

$\therefore K \propto 1/r$ ——————(i)

→ According to the Kepler's third law,

$r \propto T^{2/3}$ ——————(ii)

from equation (i) and (ii)

$K \propto T^{-2/3}$

(183) The density of a planet is. What is the orbital period of a satellite revolving around it; remaining close to its surface ?

(A) $(3\pi / G\ \ )^{3/2}$ **(B) $(3\pi / G\ \ )^{1/2}$**

(C) $(3\pi / 2G\ \ )^{1/2}$ **(D) $(3\pi / 2G\ \ )^{3/2}$**

Solution :

$$\rightarrow \quad \frac{mv^2}{R} = \frac{GMm}{R^2}$$

$$\rightarrow \quad v = \left(\frac{GM}{R}\right)^{\frac{1}{2}}$$

$$\rightarrow \quad R\omega = \left(\frac{GM}{R}\right)^{\frac{1}{2}} \quad (\therefore v = R\omega)$$

→

$$R\omega = \left(\frac{G \times \frac{4}{3}\pi R^3 \rho}{R}\right)^{\frac{1}{2}} \qquad \left(\because M = \frac{4}{3}\pi R^3 \rho\right)$$

$$\rightarrow \quad \frac{2\pi}{T} = \left(\frac{4}{3}\pi\, GR^2 \rho\right)^{\frac{1}{2}} \times \frac{1}{R} \qquad \left(\because \omega = \frac{2\pi}{T}\right)$$

$$\rightarrow \quad \frac{2\pi}{T} = \left(\frac{4}{3}\pi\, G\rho\right)^{\frac{1}{2}}$$

$$\rightarrow \quad T = \frac{2\pi}{\left(\frac{4}{3}\pi\, G\rho\right)^{\frac{1}{2}}}$$

Ans **: (181) B (182) A (183) B**

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$$\therefore \quad T = \left(\frac{3\pi}{G\rho}\right)^{\frac{1}{2}}$$

(184) An artificial satellite revolves around the Earth, remaining close to the surface of the Earth. Show that its time-period is $T = 2\pi\sqrt{\frac{R_e}{g}}$

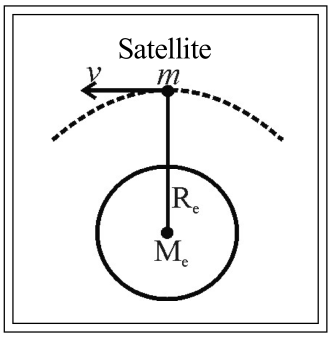
Ans :

$\rightarrow$ Put $r = R_e$ directly in $T^2 = \left(\frac{4\pi^2}{GM_e}\right)$

$$T^2 = \left(\frac{4\pi^2 R_e^2}{GM_e}\right) R_e$$

$$= \left(\frac{4\pi^2}{g}\right) R_e$$

$$\therefore \quad T = 2\pi\sqrt{\frac{R_e}{g}}$$



(185) A geo-stationary satellite orbits around the earth in a circular orbit of radius 36000 km. Then, the time period of a spy satellite orbiting a few hundred km above the earth's surface (R_{earth} = 6400 km) will approximately be............. [IIT-2002]

(A) 1/2 hr (B) 1 hr (C) 2 hr (D) 4 hr

(186) Using orbital radius r and the corresponding periodic time T of different satellites revolving around a planet, what would be the slope of the graph of log r $\rightarrow$ log T ?

(A) 3/2 (B) 3 (C)2/3 (D) 2

Solution :

$\Rightarrow$ From,

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3;$$

$$\Rightarrow r^3 = \left(\frac{GM}{4\pi^2}\right) T^2$$

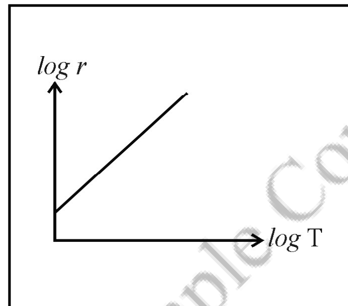
$$\therefore \quad r = \left(\frac{GM}{4\pi^2}\right)^{1/3} T^{2/3}$$

$$\Rightarrow \quad \log r = (1/3)\log\left(\frac{GM}{4\pi^2}\right) + (2/3)\log T$$

$$= (2/3)\log T + (1/3)\log\left(\frac{Gm}{4\pi^2}\right)$$

comparing with $y = mx + C$

slope of the graph m = (2/3)



(187)For different satellites revolving around a planet in different circular orbits, which of the following shows the relation between the angular momentum L and the orbital radius r ?

(A) $L \propto \frac{1}{\sqrt{r}}$ (B) $L \propto r^2$

(C) $L \propto \sqrt{r}$ (D) $L \propto \frac{1}{r^2}$

Solution :

$$\Rightarrow \quad L = mvr = m\left(\sqrt{\frac{GM_e}{r}}\right) r$$

$$L = m\sqrt{GM_e}\ \sqrt{r}$$

$$L \propto \sqrt{r}$$

(188) A satellite of mass m revolves around the Earth in the circular orbit of radius r. What is its angular momentum ? The mass of the earth is M.

(A) $m\sqrt{\frac{Gm}{r}}$ (B) $M\sqrt{Gmr}$

(C) $M\sqrt{\frac{Gm}{r}}$ (D) $m\sqrt{GMr}$

Ans **: (185) C (186) C (187) C (188) D**

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Solution :

→ Angular momentum of satellite having mass m,

$L = mvr$

$L = m\left(\sqrt{\frac{GM}{r}}\right)r \quad \left(\because \text{Orbital speed of satellite } v = \sqrt{\frac{GM}{r}}\right)$

$= m\sqrt{GMr}$

(189) A satellite of mass m revolves in a circular orbit of radius r around a planet of mass M. What is its areal velocity ?

(A) $\sqrt{Gmr}/2$ (B) $\sqrt{GMr}/2$

(C) $\sqrt{Gmr}$ (D) $m\sqrt{GMr}$

Solution :

Areal velocity of satellite $\frac{dA}{dt} = \frac{L}{2m}$

$\therefore \frac{mvr}{2m} = \frac{vr}{2}$

$= \frac{1}{2}\left(\sqrt{\frac{GM}{r}}\right) \times r$

$\left(\because \text{Orbital speed of satellite } v = \sqrt{\frac{GM}{r}}\right)$

$\therefore \frac{dA}{dt} = \frac{1}{2}\sqrt{GMr}$

(190) A binary star system consists of two stars A and B which have time period T_A and T_B, radius R_A and R_B and mass M_A and M_B. Then..... [IIT-2006]

(A) if $T_A > T_B$ then $R_A > R_B$

(B) if $T_A > T_B$ then $M_A > M_B$

(C) $(T_A/T_B)^2 = (R_A/R_B)^3$

(D) $T_A = T_B$

(191) An artificial satellite moving in a circular orbit around the Earth has a total (kinetic + potential) energy E_0. Its potential energy is... [IIT-1997]

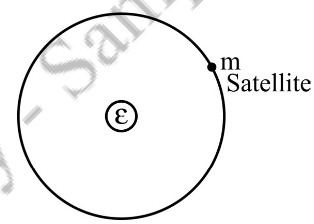
(A) - E_0 (B) 1.5 E_0 (C) 2E_0 (D) E_0

(192) The satellite of mass m is orbiting around the earth in a circular orbit with a velocity *v*. What will be its total energy ? [NEET-1991]

(A) $\frac{3}{4}mv^2$ (B) $\frac{1}{2}mv^2$

(C) mv^2 (D) $-(\frac{1}{2})mv^2$

Solution :



Total energy of satellite = P.E + K.E.

$= -\frac{GMm}{R} + \frac{1}{2}mv^2$

$= -\frac{GMm}{R} + \frac{m}{2}\left(\frac{GM}{R}\right) \quad \left(\because v = \sqrt{\frac{GM}{R}}\right)$

$= -\frac{GMm}{2R}$

$\therefore$ Total energy of satellite $= -\frac{1}{2}mv^2$

(193) Choose the correct alternative :

(a) If the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of its kinetic /potential energy.

(b) The energy required to launch an orbiting satellite out of earth's gravitational influence is more / less

***Ans* : (189) B (190) D (191) C (192) D**

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than the energy required to project a stationary object at the same height (as the satellite) out of earth's influence.

Solution :

(a) Kinetic energy

(b) Less

(194) A roller coaster is designed such that riders experience "weightlessness" as they go round the top of a hill whose radius of curvature is 20 m. The speed of the car at the top of the hill is between [NEET-2008]

(A) 14 *m/s* and 15 *m/s*
(B) 15 *m/s* and 16 *m/s*
(C) 16 *m/s* and 17 *m/s*
(D) 13 *m/s* and 14 *m/s*

Solution :

The appearance of weightlessness occurs in space when the gravitational attraction of the earth on a body in space is equal to the centripetal force.

$$\frac{mv^2}{r} = mg$$

$$\therefore v = \sqrt{rg}$$

$$= \sqrt{20 \times 10}$$

$$\boxed{v = 14.14 \text{ m/s}}$$

(195) What kind of relation exists between the kinetic energy (E_k) and the orbital radius (r) of the satellites revolving around the Earth?

(A) $E_k \propto r$ (B) $E_k \propto (1/r)$
(C) $E_k \propto r^2$ (D) $E_k \propto (1/r^2)$

Solution :

$\Rightarrow$ gravitational acceleration for rotation of planet = gravitational force

$$\frac{mv^2}{r} = \frac{GM_em}{r^2} \quad \therefore \frac{1}{2}mv^2 = \frac{GM_em}{2r}$$

$$\therefore \quad \text{In } E_k = \left(\frac{GM_em}{2}\right)\cdot\frac{1}{r}$$

$$\frac{GM_em}{2} \text{ constant}$$

$$\therefore \quad E_k \propto \frac{1}{r}$$

Notes

Ans **: (194) A (195) B**

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Current Topic Practice

(196) If the gravitational force between two objects were proportional to 1/R (and not as $1 / R^2$), where R is separation between them, then a particle in circular orbit under such a force would have its orbital speed v proportional to [NEET-1989]

(A) $1 / R^2$ (B) R^0 (C) R (D) 1 / R

(197) A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, The orbital speed of the satellite is.... [AIIEE-2004]

(A) $\frac{gR^2}{R+x}$ (B) $\frac{gR}{R-x}$ (C) gx (D) $\left(\frac{gR^2}{R+x}\right)^{1/2}$

(198) Suppose there existed a planet that went around the sun twice as fast as the earth. What would be its orbital size as compared to that of the earth ?

(199) The period of revolution of planet A round the sun is 8 times that of B. The distance of A from the sun is how many times greater than that of B from the sun ? [NEET-1997]

(A) 5 (B) 4 (C) 3 (D) 2

(200) The distances of two planets from the sun are $10^{13} m$ and $10^{12} m$ respecitvely. The ratio of time periods of these two planets is [NEET-1988]

(A) $\frac{1}{\sqrt{10}}$ (B) 100 (C) $10\sqrt{10}$ (D) $\sqrt{10}$

(201) A geostationary satellite is orbiting the earth at a height of 6 R from the earth's surface (R is the earth's radius). What is the period of rotation of another satellite at a height of 2.5 R from the earth's surface [AIIMS-2011]

(A) $6\sqrt{2}$ hours

(B) 10 hours

(C) ($5\sqrt{5}$ / 3) hours

(D) none of the above

(202) A satellite revolves around the Earth at a height from surface equal to the radius of the Earth. Calculate its
(1) orbital speed (2) time period.
Take G = 6.67×10^{-11} Nm^2/kg^2,
raidus of the Earth = 6400 km and
mass of the Earth = 6×10^{24} kg.

(203) How will you 'weigh the sun', that is estimate its mass? The mean orbital radius of the earth around the sun is 1.5×10^8 km.

(204) A satellite is launched into a circular orbit of radius R around the earth. While a second satellite launched into an orbit of radius 1.01 R. The period of the second satellite is longer than the first one by approximately : [AIIMS-2002]

(A) 3.0 % (B) 1.5 %

(C) 0.7 % (D) 1.0 %

(205) For a satellite moving in an orbit around the earth, the ratio of kinetic energy to potential energy is .... [NEET-2005]

(A) 2 (B) $\frac{1}{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}$

***Ans* : (196) B (197) B (199) B (200) C (201) A (204) B (205) B**

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(206) Two satellites of earth, S_1 and S_2, are moving in the same orbit. The mass ofS_1 is four times the mass S_2. Which one of the following statements is true ? [NEET-2007]

(A) The time period of S_1 is four times that of S_2

(B) The potential energies of earth and satellite in the two cases are equal.

(C) S_1 and S_2 are moving with the same speed.

(D) The kinetic energies of two satellites are equal.

(207) The geostationary satellite revolves .....

(A) from south to north in the polar plane

(B) from north to south in the polar plane

(C) from east to west in the equatorial plane

(D) from west to east in the equatorial plane

(208) Make suitable pairs :

I	II
(P) polar satellite (Q) Geostationary satellite	(1) tele communication (2)useful in moon expendition (3) spying

(A) P → 1, Q → 2 (B) P → 3, Q → 1

(C) P → 3, Q → 2 (D) P → 2, Q → 3

(209) A ball is dropped from a satellite revolveing around the earth at a height of 120 km. The ball will....... [NEET-1996]

(A) continue to move with same speed along a straight line tangentially to the satellite at that time

(B) continue to move with the same speed along the original orbit of satellite

(C) fall down to earth gradually

(D) go far away in space

(210) The planet Mars has two moons, phobos and delmos.

(i) phobos has a period 7 hours, 39 minutes and an orbital radius of 9.4×10^3 km. Calculate the mass of mars.

(ii) Assume that earth and mars move in circular orbits around the sun, with the martian orbit being 1.52 times the orbital radius of the earth. What is the length of the martian year in days ?

(211) Weighing the Earth : You are given the following data: $g = 9.81$ ms^{-2}, $R_E = 6.37 \times 10^6$ m, the distance to the moon $R = 3.84 \times 10^8$ m and the time period of the moon's revolution is 27.3 days. Obtain the mass of the Earth M_E in two different ways.

(212) A 400 kg satellite is in a circular orbit of radius $2R_E$ about the Earth. How much energy is required to transfer it to a circular orbit of radius $4R_E$? What are the changes in the kinetic and potential energies ?

(213) If suddenly the gravitationl force of attraction between Earth and a satellite revolving around it becomes zero, then the satellite will [AIEEE-2002]

(A) Continue to move in its orbit with same velocity

(B) move tangentially to the original orbit in the same velocity

(C) become stationary in its orbit

(D) move towards the earth

***Ans* : (206) C (207) D (208) B (209) B (213) B**

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(214) The time period of an earth satellite in circular orbit is independent of [AIEEE-2004]

(A) the mass of the satellite

(B) radius of its orbit

(C) both the mass and radius of the orbit

(D) neither the mass of the satellite nor the radius of its orbit.

(215) A satellite of mass m is orbiting the earth (of radius R) at a height h from its surface. The total energy of the satellite in terms of g_0, the value of acceleration due to gravity at the earth's surface, is : [NEET : PHASE : 2 (2016)]

(A) $\frac{2mg_0R^2}{R+h}$ (B) $-\frac{2mg_0R^2}{R+h}$

(C) $\frac{mg_0R^2}{2(R+h)}$ (D) $-\frac{mg_0R^2}{2(R+h)}$

(216) The mean radius of earth is R, its angular speed on its own axis is ω and the acceleration due to gravity at earth's surface is g. What will be the radius of the orbit of a geostationary satellite ? [NEET-1992]

(A) $(R^2g/\omega^2)^{1/3}$ (B) $(Rg/\omega^2)^{1/3}$

(C) $(R^2\omega^2/g)^{1/3}$ (D) $(R^2g/\omega)^{1/3}$

(217) The escape velocity from earth is 11.2 km/s. If a body is to be projected in a direction making an angle 45^0 to the vertical, then the escape velocity is.... [NEET-1993]

(A) 11.2 × 2 *km/sec* (B) 11.2 *km/sec*

(C) $\frac{11.2}{\sqrt{2}}$ *km/sec* (D) $11.2\sqrt{2}$ *km/sec*

(218) A satellite A of mass m is at a distance r from the centre of the earth. Another satellite B of mass 2m is at a distance of 2r from the earth's centre. Their time periods are in the ratio of.... [NEET-1993]

(A) 1 : 2 (B) 1 : 16

(C) 1 : 32 (D) $1 : 2\sqrt{2}$

(219) A geostationary satellite is orbiting the earth at a height of 6 R from the earth's surface (R is the earth's radius). What is the period of rotation of another satellite at a height of 2.5 R from the earth's surface [AIIMS-2011]

(A) $6\sqrt{2}$ hours

(B) 10 hours

(C) $(5\sqrt{5}\ /\ 3)$ hours

(D) none of the above

(220) A satellite is launched into a circular orbit of radius R around the earth. While a second satellite launched into an orbit of radius 1.01 R. The period of the second satellite is longer than the first one by approximately : [AIIMS-2002]

(A) 3.0 % (B) 1.5 %

(C) 0.7 % (D) 1.0 %

Notes

***Ans* : (214) A (215) D (216) A (217) B (218) D (219) A (220) B**

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Hints & Solutions

196.

→ Gravitational force between two objects

$$F \propto \frac{1}{R} \quad \therefore F \propto \frac{k}{R}\left(\therefore\ k = \text{Constant}\right)$$

→ In equilibrium, the gravitational force provides the required centripetal force to the partical.

$$\frac{mv^2}{R} = \frac{k}{R}$$

$$\therefore \boxed{v \propto R^o}$$

197.

→ Centripital force of a satellite = gravitational force of earth

$$\rightarrow \frac{mv_o^2}{R+x} = \frac{GMm}{(R+x)^2}$$

$$\therefore v_o^2 = \frac{GM}{R+x}$$

$$= \frac{gR^2}{R+x} \qquad \left(\because g = \frac{GM}{R^2}\right)$$

$$\boxed{\therefore v_o = \left(\frac{gR^2}{R+x}\right)^{1/2}}$$

198.

→ Let period of revolution of the earth = T_e

period of revolution of the planet = T_p

$$T_P = \frac{T_e}{2}$$

→ Orbital size of the earth r_e = 1 AU

Orbital size of the planet r_p = ?

From kepler's third law

$$\frac{T_P^2}{T_e^2} = \frac{r_P^3}{r_e^3}$$

$$r_P = \left(\frac{T_P}{T_e}\right)^{2/3} r_e$$

$$= \left(\frac{T_2/2}{T_e}\right)^{2/3} \times 1\ \text{AU}$$

$$= (0.5)^{2/3}\ \text{AU}$$

$$r_P = 0.63\ \text{AU}$$

199.

→ According to kepler's third law,

$$T^2 \propto r^3$$

$$\therefore \frac{T_A^2}{T_B^2} = \frac{R_A^3}{r_B^3}$$

$$\therefore \frac{r_A}{r_B} = \left(\frac{T_A}{T_B}\right)^{2/3}$$

$$= (8)^{2/3}$$

$$= 4$$

$$\boxed{r_A = 4\ r_B}$$

200.

→ According to kepler's third law,

$$T^2 \propto r^3$$

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3} = \frac{(10^{13})^3}{(10^{12})^3}$$

$$= 10^{39-36}$$

$$= 10^3$$

$$\therefore \frac{T_1}{T_2} = 10\sqrt{10}$$

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201.

→ $T^2 \propto r^3$

→ $\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$

$= \left(\frac{7\text{ R}}{3.5\text{ R}}\right)^{3/2}$

→ $\frac{T_1}{T_2} = \sqrt{8}$

→ $T_2 = \frac{T_1}{\sqrt{8}}$

$= \frac{24}{\sqrt{8}} = 6\sqrt{2}\text{ h}$

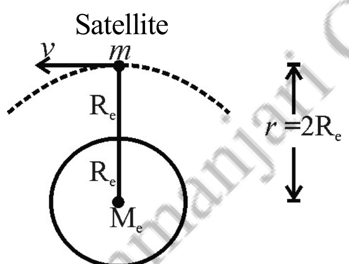
$\boxed{T_2 = 6\sqrt{2}\text{ h}}$

202.

→ G $= 6.67 \times 10^{-11}$ Nm²/kg²

$R_e = 6400$ km $= 64 \times 10^5$ m

$M_e = 6 \times 10^{24}$ kg



(*i*) centripetal force = gravitational force

$$\frac{mv^2}{r} = \frac{GM_em}{r^2}$$

$$v^2 = \frac{GM_e}{2R_e} \quad (\because r = 2R_e)$$

$$= \frac{(6.67 \times 10^{-11})(6 \times 10^{24})}{2(64 \times 10^5)}$$

$$= 0.3126 \times 10^8$$

$$\boxed{v = 5591\ m/s}$$

(*ii*) $v = r\omega$

$$\omega = \frac{v}{r}$$

$$\frac{2\pi}{T} = \frac{v}{2R_e} \quad (\because \omega = 2\pi/T,\ r = 2R_e)$$

$$T = \frac{4\pi R_e}{v}$$

$$T = \frac{4(3.14)(64 \times 10^5)}{5591}$$

$$\boxed{T = 1.437 \times 10^4\ sec \approx 14370\text{ s}}$$

203.

→ $r = 1.5 \times 10^8$ km $= 1.5 \times 10^{11}$ m

→ T = 365 days = 365 × 24 × 3600 sec.

→ Let the mass of the sun is M

Centrilpetal force required between the eath and sun = force of gravitation

$$\therefore \frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$\therefore \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{GMm}{r^2}$$

$$\therefore M = \frac{4\pi^2 r^3}{GT^2}$$

$$= \frac{4 \times 9.87 \times (1.5 \times 10^{11})^3}{6.67 \times 10^{-11}\ (365 \times 24 \times 3600)^2}$$

$M = 2.01 \times 10^{30}$ kg

204.

$T \propto r^{3/2}$

→ $T \propto \sqrt{r}$ Whatever percentage increases in r half of the percentage will increase in T.

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∴ From $T \propto r^3$, whatever percentage increases in r, its 3 times of the percentage will increase in T here r increases by 1% so T will be increases by 3%

Here $T \propto \sqrt{r}\, r^3$ so T will become (0.5%) (3%) = 1.5%

210.

(*i*) We employ $T^2 = k\,(R_E + h)^3$
[where $k = (4\pi^2 / GM_E)$] with M_E replaced by the martian mass M_m

$$T^2 = \frac{4\pi^2}{GM_E} R^3$$

$$M_m = \frac{4\pi^2}{G}\frac{R^3}{T^2}$$

$$= \frac{4 \times (3.14)^2 \times (9.4)^3 \times 10^{18}}{6.67 \times 10^{-11} \times (4.59 \times 60)^2}$$

$$M_m = \frac{4 \times (3.14)^2 \times (9.4)^3 \times 10^{18}}{6.67 \times (4.59 \times 6)^2 \times 10^{-5}}$$

$$= 6.48 \times 10^{23}\, kg$$

(*ii*) Once again Kepler's third law comes to our aid,

$$\frac{T_M^2}{T_E^2} = \frac{R_{MS}^3}{R_{ES}^3}$$

where R_{MS} is the mars -sun distance and R_{ES} is the earth-sun distance.

$\therefore T_M = (1.52)^{3/2} \times 365$

$= 684$ days

→ We note that the orbits of all planets except Mercury, Mars and Pluto are very close to being circular. For example, the ratio of the semiminor to semi-major axis for our Earth is, $(b / a) = 0.99986$.

211.

→ From $g = \dfrac{F}{m} = \dfrac{GM_E}{R_E^2}$

We have $M_E = \dfrac{gR_E^2}{G}$

$$= \frac{9.81 \times (6.37 \times 10^6)^2}{6.67 \times 10^{-11}}$$

$$= 5.97 \times 10^{24}\, kg$$

The moon is a satellite of the Earth. From the derivation of Kepler's third law

$T^2 = k\,(R_E + h)^3 \left[\text{where } k = \dfrac{4\pi^2}{GM_E}\right]$

$$T^2 = \frac{4\pi^2 R^3}{GM_E}$$

$$M_E = \frac{4\pi^2 R^3}{GT^2}$$

$$= \frac{4 \times (3.14)^2 \times (3.84)^3 \times 10^{24}}{6.67\times10^{-11} \times (27.3\times24\times60\times60)^2}$$

$$= 6.02 \times 10^{24}\, kg$$

→ Both methods yield almost the same answer, the difference between them being less than 1%.

212.

→ Initially, $E_i = -\dfrac{GM_E m}{4R_E}$

→ While finally $E_f = -\dfrac{GM_E m}{8R_E}$

→ The change in the total energy is

$\Delta E = E_f - E_i$

$$= \frac{GM_E m}{8R_E} = \left[\frac{GM_E}{R_E^2}\right]\frac{mR_E}{8}$$

$$\Delta E = \frac{gmR_E}{8} = \frac{9.81 \times 400 \times 6.37 \times 10^6}{8}$$

$$\Delta E = 3.13 \times 10^9\, J$$

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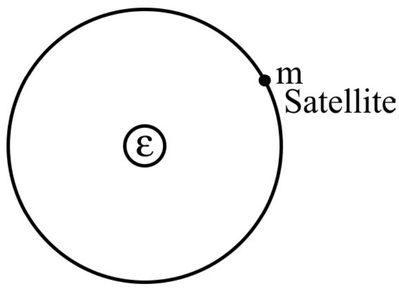
→ The kinetic energy is reduced and it mimics ΔE, namely,

$\Delta K = K_f - K_i = -3.13 \times 10^9$ J.

→ The change in potential energy is twice the change in the total energy, namely

$\Delta V = V_f - V_i = -6.25 \times 10^9$ J

213.



214.

We know that,

$$r^3 = \frac{GM_eT^2}{4\pi^2}$$

$$\rightarrow \quad T = \left(\frac{4\pi^2 r^3}{GM_e}\right)^{1/2}$$

From above equation time period of an earth satellite is independant of its mass.

215.

→ Total energy of a satellite at height h

= K.E. + P.E.

$$= \frac{GMm}{2(R+h)} - \frac{GMm}{(R+h)}$$

$$= \frac{-GMm}{2(R+h)}$$

$$= \frac{-GMmR^2}{2R^2(R+h)}$$

$$= \frac{-mg_oR^2}{2(R+h)} \qquad \left(\because g_o = \frac{GM}{R^2}\right)$$

216.

→ Orbital speed of satellite is $v_o = \sqrt{\frac{GM}{r}}$

$$v_o = \sqrt{\frac{gR^2}{r}}$$

→ Periodic time

$$T = \frac{2\pi r}{v_0} \qquad \left(\because g = \frac{GM}{R^2}\right)$$

$$= \frac{2\pi r}{\left(\frac{gR^2}{r}\right)^{1/2}}$$

$$= \frac{2\pi r^{3/2}}{\sqrt{gR^2}}$$

but $T = 2\pi/\omega$

$$\rightarrow \quad T = \frac{2\pi r^{3/2}}{\sqrt{gR^2}} = \frac{2\pi}{\omega}$$

$$\text{Hence } r^{3/2} = \frac{\sqrt{gR^2}}{\omega}$$

$$\therefore r^3 = \frac{gR^2}{\omega^2}$$

$$\therefore \boxed{r = \left(\frac{gR^2}{\omega^2}\right)^{1/3}}$$

217.

→ Escape speed is independant from angle of projection.

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218.

→ From kepler's third law,

$$T^2 \propto r^3$$

$$\frac{T_1}{T_2} = \frac{r^3}{(2r)^3} = \frac{r^3}{8r^3}$$

$$\therefore \frac{T_1}{T_2} = \frac{1}{2\sqrt{2}}$$

219.

→ $T^2 \propto r^3$

→ $\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$

$= \left(\frac{7R}{3.5\ R}\right)^{3/2}$

→ $\frac{T_1}{T_2} = \sqrt{8}$

→ $T_2 = \frac{T_1}{\sqrt{8}}$

$= \frac{24}{\sqrt{8}} = 6\sqrt{2}\ h$

$\boxed{T_2 = 6\sqrt{2}\ h}$

220.

$T \propto r^{3/2}$

→ From $T \propto \sqrt{r}$, whatever percentage increases in r half of the percentage will increase in T. So 1% here r increases by 1% so T will be increases by 0.5%.

→ From $T \propto r^3$, whatever percentage increases in r, its 3 times of the percentage will increase in T.

∴ Here r increases by 1% so T will be increases by 3%.

Here $T \propto \sqrt{r}\ r^3$

so T will become (0.5%) (3%) = 1.5%.

Notes

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Miscellaneous Problems

(221) Prove that the ratio of the rate of change of g at a height equal to the Earth's radius from the surface of the Earth to the value of g at the surface of the Earth is equal to -1/4R_e

Ans :

→ The gravitational acceleration at distance $r \geq R_e$ from the center of the Earth is $g(r) = GM_e / r^2$

→ Differentiating with respect to r,

$$\left[\frac{dg(r)}{dr}\right] = \frac{-2GM_e}{r^3}$$

and $r = R_e + h = R_e + R_e = 2R_e$

$$\therefore \left[\frac{dg(r)}{dr}\right]_{2R_e} = \frac{-2GM_e}{(2R_e)^3} = \frac{-2GM_e}{8R_e^{\ 3}}$$

→ But gravitational acceleration on the surface of the Earth $g_e = \frac{GM_e}{R_e^{\ 2}}$

$$\frac{\left[\frac{dg(r)}{dr}\right]_{2R_e}}{g_e} = \frac{-2GM_e}{8R_e^{\ 3}} \times \frac{R_e^{\ 2}}{GM_e} = \frac{-1}{4R_e}$$

(222) Two satellites S_1 and S_2 revolve around a planet in two different coplanar circular orbits in the same direction their periods are 31.4 h and 62.8 h and the radius orbit of S_1 is 4000 km, find

(i) the radius of the orbit of S_2

(ii) the magnitudes of the velocities of the two satellites.

Ans :

(i) $T^2 \propto r^3$

$$\therefore \frac{T_1^{\ 2}}{T_2^{\ 2}} = \frac{r_1^{\ 3}}{r_2^{\ 3}}$$

$$\therefore r_2^{\ 3} = r_1^{\ 3}\left(\frac{T_2^{\ 2}}{T_1^{\ 2}}\right)$$

$$= (4000)^3\left(\frac{62.8^2}{31.4^2}\right)$$

$$\therefore r_2 = (4000)(4)^{1/3} = (4000)(1.588)$$

$$\boxed{r_2 = 6352\ km}\ (✪)$$

(ii)

$$v_1 = \frac{2\pi r_1}{T_1} = \frac{(2)(3.14)(4000)}{31.4}$$

$$\boxed{v_1 = 800\ km/h}$$

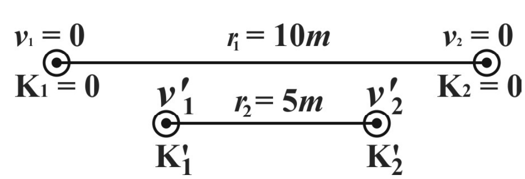
$$v_2 = \frac{2\pi r_2}{T_2} = \frac{(2)(3.14)(6352)}{62.8}$$

$$\boxed{v_2 = 635.2\ km/h}$$

(223) Two objects of masses 1 kg and 2 kg respectively are released from rest when their separation is 10 m. Assuming that only mutual gravitational force act on them, find the velocity of each of them when separation becomes 5m.

(Take $G = 6.66 \times 10^{-11}\ Nm^2 / kg^2$)

Ans :



(✪) $x = 4^{1/3} \Rightarrow log x = (1/3)\ log 4 = (1/3)\ (0.6021)$

$log x = 0.2007 \Rightarrow x = Antilog\ (0.20007)$

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→ Initially velocities of both the particles are zero and hence their kinetic energies are zero. (i.e. $v_1 = v_2 = 0$; $K_1 = K_2 = 0$)

→ When the separtation is 5m, their velocities are v_1' and v_2' and kinetic energis are K_1' and K_2' Respectively.

→ For this system initial potential energy

$$U_1 = \frac{-G\, m_1 m_2}{r_1}$$

$$= \frac{-(6.67 \times 10^{-11})\,(1 \times 2)}{10}$$

$$= -13.32 \times 10^{-12}\ J$$

→ Final potential energy $U_2 = \dfrac{-G\, m_1 m_2}{r_2}$

$$= \frac{-(6.67 \times 10^{-11})\,(1 \times 2)}{5}$$

$$= -26.64 \times 10^{-12}\ J$$

∴ Change in potential energy

$$\Delta U = U_2 - U_1$$

$$= -26.64 \times 10^{-12} - (-13.32 \times 10^{-12})$$

$$= -13.32 \times 10^{-12}\ J$$

→ According to the law of conservation of mechanical energy

$K + U =$ constant ∴ $\Delta K + \Delta U = 0$.

∴ $\Delta K = -\Delta U$

∴ $(K_1' + K_2') - 0 = -(U_2 - U_1)$

∴ $(\frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2) - (0)$

$= 13.32 \times 10^{-12}$ J(♠)

∴ $\dfrac{v_1'^2}{2} + v_2'^2 = 13.32 \times 10^{-12}$ ——— (1)

→ (✩) According to the law of conservation of momentum, final total momentum = initial total momentum.

(♠) By taking $m_1 = 1$kg, $m_2 = 2$ kg
(✩) Total momentum will be zero because, external force is zero.

∴ $0 = m_1\vec{v}_1' + m_2\vec{v}_2'$

∴ $m_1\vec{v}_1' = -m_2\vec{v}_2'$

∴ $\vec{v}_1' = -\dfrac{m_2}{m_1}\vec{v}_2'$

∴ $|\vec{v}_1'| = \left(\dfrac{m_2}{m_1}\right)(|\vec{v}_2'|)$

∴ $v_1' = 2\, v_2'$ ——— (2)

→ From eqns. (1) and (2)

$$\frac{4v_1'^2}{2} + v_2'^2 = 13.32 \times 10^{-12}$$

∴ $3v_2'^2 = 13.32 \times 10^{-12}$

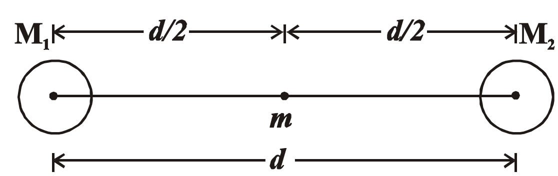
∴ $v_2'^2 = 4.44 \times 10^{-12} = 444 \times 10^{-14}$

∴ $\boxed{v_2' = 21.07 \times 10^{-7}\ m/s}$

∴ $\boxed{v_1' = 42.14 \times 10^{-7}\ m/s}$

(224) The mass and radius of the Earth are M_1, R_1 and those for the moon are M_2, R_2 respectively. The distance between their centres is d. With what velocity should an object of mass m be thrown away from the mid-point of the line joining them so that it escapes to infinity ?

Ans :



→ The distance between the Earth and moon is d. The 'm' is situated at $\frac{d}{2}$ from Earth and moon.

P.E. of the mass ' m' due to Earth is

$$U_1 = -\frac{GM_1 m}{d/2} = -\frac{2GM_1 m}{d} \quad \text{——— (1)}$$

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→ Let the potential energy of mass m due to moon at distance $\frac{d}{2}$ be

$$U_2 = -\frac{GM_2 m}{d/2} = -\frac{2GM_2 m}{d} \qquad (2)$$

→ Total Gravitational P.E., $U = U_1 + U_2$

$$= -\frac{2Gm}{d}(M_1 + M_2) \qquad (3)$$

→ Escaped energy applied to the object for sending it at infinite distance.

$$= +\frac{2Gm}{d}(M_1 + M_2)$$

→ If the particle have escape speed becomes v_e then

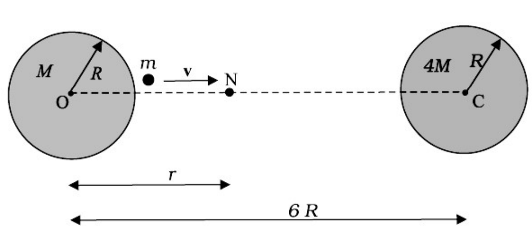
$$\frac{1}{2} m v_e^2 = \frac{2Gm}{d}(M_1 + M_2)$$

$$\therefore \boxed{v_e = 2\sqrt{\frac{G}{d}(M_1 + M_2)}}$$

Note : Find the escape velocity for a particle lying at a distance d/3 from the surface of the Earth. $d_1 = d/3$, $d_2 = 2d/3$

Ans. : $\sqrt{(3G/d)(2M_1 + M_2)}$

(225) Two uniform solid spheres of equal radii R, but mass M and 4 M have a center to centre separation 6 R, as shown in Figure. The two spheres are held fixed. A projectile of mass m is projected from the surface of the sphere of mass M directly towards the centre of the second sphere. Obtain an expression for the minimum speed v of the projectile so that it reaches the surface of the second sphere.



Solution :

→ The projectile is acted upon by two mutually opposing gravitational forces of the two spheres. The neutral point N (see Figure) is defined as the position where the two forces cancel each other exactly.

If ON = r, we have

$$\frac{G\,M\,m}{r^2} = \frac{4\,G\,M\,m}{(6R - r)^2}$$

$(6R - r)^2 = 4r^2$

$6R - r = \pm 2r$

$r = 2R$ or $-6R$.

→ The neutral point $r = -6R$ does not concern us in this example. Thus ON = r = 2R. It is sufficient to project the particle with a speed which would enable it to reach N.

→ Thereafter, the greater gravitational pull of 4M would suffice. The mechanical energy at the surface of M is

$$E_i = \frac{1}{2} m v^2 - \frac{G\,M\,m}{R} - \frac{4\,G\,M\,m}{5R}.$$

→ At the neutral point N, the speed approaches zero. The mechanical energy at N is purely potential.

$$E_N = -\frac{G\,M\,m}{2R} - \frac{4\,G\,M\,m}{4R}.$$

→ From the principle of conservation of mechanical energy

$$\frac{1}{2} v^2 - \frac{GM}{R} - \frac{4GM}{5R} = -\frac{GM}{2R} - \frac{GM}{R}$$

OR

$$v^2 = \frac{2GM}{R}\left(\frac{4}{5} - \frac{1}{2}\right)$$

$$v = \left(\frac{3GM}{5R}\right)^{\frac{1}{2}}$$

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(226) The planet Mars has two moons, phobos and delmos.
(i) phobos has a period 7 hours, 39 minutes and an orbital radius of 9.4×10^3 km. Calculate the mass of mars.
(ii) Assume that earth and mars move in circular orbits around the sun, with the martian orbit being 1.52 times the orbital radius of the earth. What is the length of the martian year in days ?

Solution :

(i) We employ $T^2 = k (R_E + h)^3$ [where $k = (4\pi^2 / GM_E)$] with M_E replaced by the martian mass M_m

$$\text{From, } T^2 = \frac{4\pi^2}{GM_m} R^3$$

$$M_m = \frac{4\pi^2}{G} \frac{R^3}{T^2}$$

$$= \frac{4 \times (3.14)^2 \times (9.4)^3 \times 10^{18}}{6.67 \times 10^{-11} \times (459 \times 60)^2}$$

$$M_m = \frac{4 \times (3.14)^2 \times (9.4)^3 \times 10^{18}}{6.67 \times (4.59 \times 6)^2 \times 10^{-5}}$$

$$= 6.48 \times 10^{23} \, kg$$

(ii) Once again Kepler's third law comes to our aid,

$$\frac{T_M^2}{T_E^2} = \frac{R_{MS}^3}{R_{ES}^3}$$

where R_{MS} is the mars -sun distance and R_{ES} is the earth-sun distance.

$\therefore T_M = (1.52)^{3/2} \times 365$

$= 684$ days

→ We note that the orbits of all planets except Mercury, Mars and Pluto are very close to being circular. For example, the ratio of the semiminor to semi-major axis for our Earth is, $(b / a) = 0.99986$.

(227) Weighing the Earth : You are given the following data: $g = 9.81 \text{ ms}^{-2}$, $R_E = 6.37 \times 10^6$ m, the distance to the moon $R = 3.84 \times 10^8$ m and the time period of the moon's revolution is 27.3 days. Obtain the mass of the Earth M_E in two different ways.

Solution :

→ From $g = \frac{F}{m} = \frac{GM_E}{R_E^2}$

we have, $M_E = \frac{gR_E^2}{G}$

$$= \frac{9.81 \times (6.37 \times 10^6)^2}{6.67 \times 10^{-11}}$$

$$= 5.97 \times 10^{24} \, kg$$

→ The moon is a satellite of the Earth. From the derivation of Kepler's third law

$$T^2 = k (R_E + h)^3 \quad \left(\text{where } k = \frac{4\pi^2}{GM_E} \right)$$

$$T^2 = \frac{4\pi^2 R^3}{GM_E}$$

$$M_E = \frac{4\pi^2 R^3}{GT^2}$$

$$= \frac{4 \times (3.14)^2 \times (3.84)^3 \times 10^{24}}{6.67 \times 10^{-11} \times (27.3 \times 24 \times 60 \times 60)^2}$$

$$= 6.02 \times 10^{24} \, kg$$

→ Both methods yield almost the same answer, the difference between them being less than 1%.

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(228) Express the constant k of $[T^2 = k(R_E + h)^3$ (where $k = 4\pi^2 / GM_E$)] in days and kilometres. Given $k = 10^{-13}\ s^2\ m^{-3}$. The moon is at a distance of 3.84×10^5 km from the earth. Obtain its time-period of revolution in days.

Solution :

→ Given

$k = 10^{-13}\ s^2\ m^{-3}$

$$= 10^{-13}\left(\frac{1}{(24\times 60\times 60)^2}\,d^2\right)\left(\frac{1}{\left(\frac{1}{1000}\right)^3 km^3}\right)$$

$$= 1.33 \times 10^{-14}\ d^2\ km^{-3}$$

→ Using eq^n $[T^2 = k(R_E + h)^3$ (where $k = 4\pi^2 / GM_E$)] and the given value of k, the time period of the moon is

$T^2 = (1.33 \times 10^{-14})(3.84 \times 10^5)^3$

$T = 27.3$ d

(229) Which of the following symptoms is likely to afflict an astronaut in space (a) swollen feet, (b) swollen face, (c) headache, (d) orientational problem.

Solution :

(A) : We know that the legs carry the weight of the body in the normal position due to gravity pull. The astronaut in space is in weightless state.

Hence, swollen feet may not affected his working.

(B) : In the conditions of weightless, the face of the astronaut excpeted to get more supply of blood. Due to it astronaut may develop swollen face. As eyes, ears, nose, mouth etc. are all embedded in the face, hence, swollen face may affect to great extent the seeing/hearing/smelling/eating/ capabilities of the astronaut in space.

(C) : Headache due to mental stress. It will perist wheather a person is an astronaut in space or he is on earth. It means headache will have the same effect on the astronaut in space as or a person on earth.

(D) : Space also has orientation. We also have the frames of reference in space.

Hence, orientation problems will affect the astronaut in space.

(230) Two stars each of one solar mass ($= 2 \times 10^{30}$ kg) are approaching each other for a head on collision. When they are a distance 10^9 km, their speeds are negligible. What is the speed with which they collide ? The radius of each star is 10^4 km. Assume the stars to remain undistorted until they collide. (Use the known value of G).

Solution :

→ Initial P.E. of the system $= -\frac{GM^2}{r}$

→ Final P.E. of the two stars $= -\frac{GM^2}{2R}$

→ The K.E. of the stars $= Mv^2$

→ gain in K.E. = loss in P.E.

→ $$Mv^2 = -\frac{GM^2}{r} + \frac{GM^2}{2R}$$

$$v^2 = GM\left(\frac{1}{2R} - \frac{1}{r}\right)$$

$$v = \sqrt{GM\left(\frac{1}{2R} - \frac{1}{r}\right)}$$

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$$v = \sqrt{(6.67 \times 10^{-11})(2 \times 10^{30})\left[\frac{1}{2 \times 10^{7}} - \frac{1}{10^{12}}\right]}$$

$v = 2.583 \times 10^{6}\ ms^{-1}$

(231) A star 2.5 times the mass of the sun and collapsed to a size of 12 km rotates with a speed of 1.2 rev. per second. (Extremely compact stars of this kind are known as neutron stars. Certain stellar objects called pulsars belong to this category). Will an object placed on its equator remain stuck to its surface due to gravity ? (mass of the sun = 2×10^{30} kg).

Solution :

→ m = 2.5 × Mass of sun

$= 2.5 \times 2 \times 10^{30}$

$= 5 \times 10^{30}$ kg

→ r = 12 km = 12×10^{3} m

→ f = 1.5 rps

→ $\omega = 2\pi f = 3\pi\ rads^{-1}$

→ If the acceleration due to gravity on the surface of the star is greater then the centrilpetal acceleration of the object. The object will remain struck to its surface.

i. e. $g > \frac{v^2}{r}$

→ $g = \frac{GM}{r^2}$

$$= \frac{(6.67 \times 10^{-11})(5 \times 10^{30})}{(12 \times 10^{3})^{2}}$$

$= 2.316 \times 10^{12}\ ms^{-2}$

→ Centripetal acceleration,

$$a_c = \frac{v^2}{r} = r\,(2\pi f)^2$$

$= 12000\ (2\pi \times 1.5)^2$

$a_c = 1.1 \times 10^{6}\ ms^{-2}$

As, $g > r\omega^2$

→ Therefore, the body will remain struck with the surface of star.

(232) A seconds pendulum is mounted in a rocket. Its period of oscillation decreases when the rocket

[NEET : 1991]

(A) comes down with uniform acceleration

(B) moves round the earth in a geostationary orbit

(C) moves up with a uniform velocity

(D) moves up with uniform acceleration

(233) A rubber ball is dropped from a height of 5 m on a planet where the acceleration due to gravity is not known. On bouncing it rises to 1.8 m. The ball loses its velocity on bouncing by a factor of

[NEET : 1998]

(A) 16 / 25 **(B) 2 / 5**

(C) 3 / 5 **(D) 9 / 25**

Solution :

***Ans* : (232) D (233) B**

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→ Potential energy = Kinetic energy

$$mgh = \frac{1}{2}mv^2$$

$$\therefore\ v = \sqrt{2gh}$$

→ If h_1 and h_2 are initial and final heights, then,

$$v_1 = \sqrt{2gh_1}\ ,\ v_2 = \sqrt{2gh_2}$$

→ Loss in velocity,

$$\Delta v = v_1 - v_2$$

$$= \sqrt{2gh_1} - \sqrt{2gh_2}$$

→ Fractional loss in velocity

$$= \frac{\Delta v}{v_1}$$

$$= \frac{\sqrt{2gh_1} - \sqrt{2gh_2}}{\sqrt{2gh_1}}$$

$$= 1 - \sqrt{\frac{h_2}{h_1}}$$

substituting the values,

$$\frac{\Delta v}{v_1} = 1 - \sqrt{\frac{1.8}{5}}$$

$$= 1 - \sqrt{0.36}$$

$$= 1 - 0.6$$

$$= 0.4$$

$$\boxed{\frac{\Delta v}{v_1} = 2/5}$$

Ans : **(234) B (235) C**

(234) A body projected vertically from the earth reaches a height equal to earth's radius before returning to the earth. The power exerted by the gravitational force is greatest.

[NEET : 2011]

(A) it remains constant all through

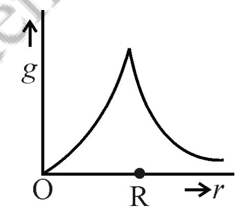
(B) at the instant just before the body hits the earth.

(C) at the instant just after the body is projected

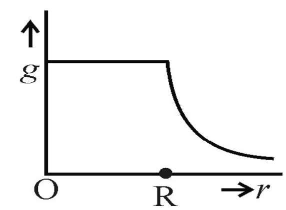
(D) at the highest position of the body

(235) Starting from the centre of the earth having radius R, the variation of g (acceleration due to gravity) is shown by [NEET : 2016]

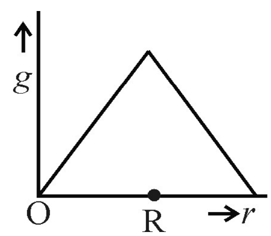
(A)



(B)



(C)



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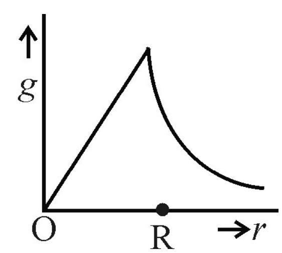

(236) Two bodies of masses m_1 and m_2 are initially at rest at infinite distance apart. They are then allowed to move towards each other under mutual gravitational attraction. Their relative velocity of approach at a separation distance r between them is :

[AIIMS-2008]

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(A) $2G\left[\frac{m_1-m_2}{r}\right]^{1/2}$

(B) $\left[\frac{2G}{r}(m_1+m_2)\right]^{1/2}$

(C) $\left[\frac{r}{2G\,(m_1m_2)}\right]^{1/2}$

(D) $\left[\frac{2G}{r}m_1m_2\right]^{1/2}$

Solution :

→ By applying law of conservation of momentum

$m_1v_1 = m_2v_2$ ... ...(i)

Where v_1 and v_2 are velocities of masses m_1 and m_2 at a distance r from each other

→ By conservation of energy,

Change in P.E. = Change in K.E.

$$\frac{Gm_1m_2}{r} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad \text{... ... ...(ii)}$$

Solving eq. (i) and (ii)

$$v_1 = \sqrt{\frac{2Gm_2^2}{r(m_1+m_2)}}$$

$$v_2 = \sqrt{\frac{2Gm_1^2}{r(m_1+m_2)}}$$

Relative velocity of approach,

$$v_R = v_1 + v_2 = \sqrt{\frac{2G}{r}\left(\frac{m_2^2+m_1^2}{m_1+m_2}\right)}$$

$$= \sqrt{\frac{2G}{r}(m_1+m_2)}$$

***Ans* :(236) B**

(237) Two satellites S_1 and S_2 revolve around a planet in the planer pane in a two different coplanar circular orbits. If its time period becomes 17.63 h and 52.9 h respectively, and the radius of S_1 becomes 6400 km, then find,

(i) orbital radius of S_2 and

(ii) the orbital velocities of both the satellites.

Solution :

→ $T^2 \propto r^3$

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

$$r_2 = \left(\frac{T_2^2 r_1^3}{T_1^2}\right)^{1/3}$$

$$= \left(\frac{52.9}{17.63}\right)^{2/3} (6400 \times 10^3)$$

r_2 = 13, 313 km

$v_1 = r_1\omega$

$$= \frac{2\pi}{T_1} r_1$$

$$= \frac{2\times 3.14\times 6400}{17.63}$$

= 2279 km/h

$$v_2 = \frac{2\pi r_2}{T_2}$$

$$= \frac{2\times 3.14\times 13313}{52.9}$$

v_2 = 1580 km/h

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(238) A satellite escaped from the earth to revolve around the Earth in a orbital radius R. Another satellite revolves in a orbital radius (1.01) R then how much time period of second satellite is more in percentage than that of the first satellite?

Solution :

→ $T^2 \propto R^3$

$$\frac{T_2}{T_1} = \sqrt{\frac{R_1^3}{R_2^3}} = \sqrt{\frac{R^3}{(1.01)^3 R^3}}$$

$$\frac{T_2}{T_1} = 0.985$$

$$\frac{T_2 - T_1}{T_1} = \frac{1 - 0.985}{1}$$

$$\frac{\Delta T}{T} \times 100 = 0.01467 \times 100 = 1.5\%$$

(239) If the total mass of a system of binary stars is M and the time period of rotation of the stars in a circular orbit about the centre of the mass of the system is T, then prove that the distance between their components is d = [(T/2π)^2 GM]

Solution :

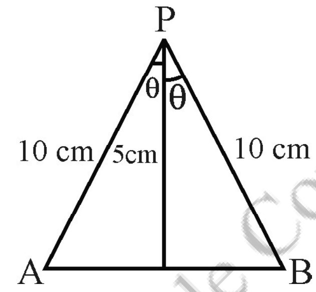
→ $$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

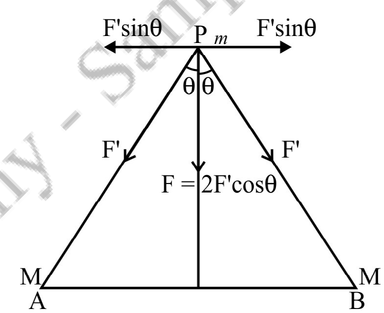
$$r^3 = \frac{GmT^2}{4\pi^2}$$

$$r = \left[\left(\frac{T}{2\pi}\right)^2 GM\right]^{1/3}$$

(240) A sphere of 10 kg is placed at each of the points A and B. What will be the initial acceleration of a small sphere placed at point P; under the effect of these two spheres only.
G = 6.67 × 10^{-11} *Nm² / kg²*



Solution :



→ Suppose F' due to sphere A

$$F' = \frac{GMm}{r^2} = \frac{6.67 \times 10^{-11} \times 10 \times m}{(10 \times 10^{-2})^2}$$

F' = 6.67 × 10^{-8} mN (P to A)

Force on mass m due to sphere B.

F' = 6.67 × 10^{-8} mN (P to B)

taking components at point p of F'.

F' sinθ components are opposite to each other so they are cancel out and F' cosθ are in same direction, so they are addup.

F = 2F' cosθ

$$= 2 \times 6.67 \times 10^{-8}\ m \times \frac{5}{10}\left[\therefore \cos\theta = \frac{5}{10}\right]$$

$$= 6.67 \times 10^{-8}\ mN$$

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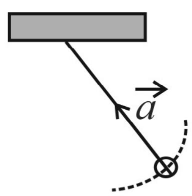
Acceleration, $a = \frac{F}{m} = \frac{6.67 \times 10^{-8}}{10 \times 10^{-3}}$

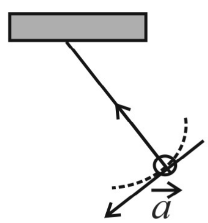
$= 6.67 \times 10^{-6}$ m s^{-2}

(241) A simple pendulum is oscillating without damping. When the displacement of the bob is less than maximum, its acceleration vector *a* is correctly shown in..... [IIT : 2002]

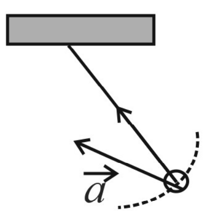
(A)



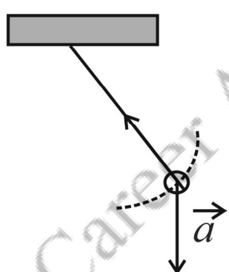
(B)



(C)



(D)



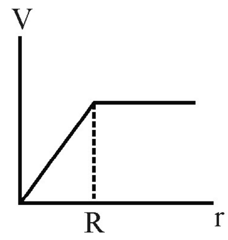
(242) A spherically symmetric gravitational system of particles has a mass density

$$= \begin{cases} \varrho_0 \text{ for } r \le R \\ 0 \text{ for } r > R \end{cases}$$

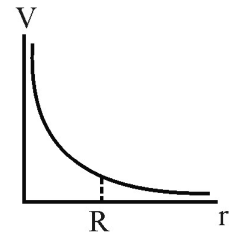
where ϱ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. It speed *v* as a function of distance r ($0 < r < \infty$) from the centre of the system is represented by.........

[IIT : 2008]

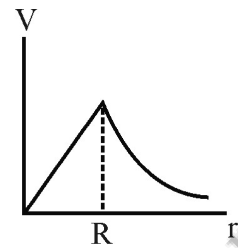
(A)



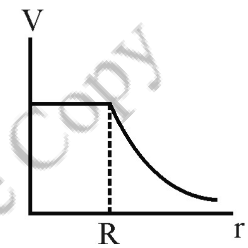
(B)



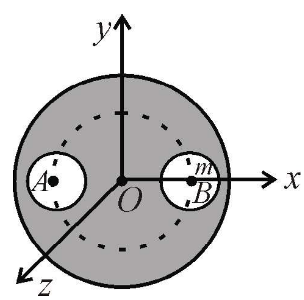
(C)



(D)



(243) A solid sphere of uniform density and radius 4 units is located with its centre at the origin O of coordinates. Two spheres of equal radii 1 unit, with their centres at A(-2, 0, 0) and B (2, 0, 0) respectively, are taken out of the solid leaving behind spherical cavities as shown in fig. [IIT : 1993]



(A) The gravitational force due to this object at the origin is zero.

(B) The gravitational force at the point B (2, 0, 0) is zero.

(C) The gravitational potential is the same at all points of circle $y^2 + z^2 = 36$.

(D) The gravitational potential is the same at all points of on the circle $y^2 + z^2 = 4$

Ans : (241) C (242) C (243) A, C, D

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PHYSICAL QUANTITIES

Sr. No.	Physical qunatity	Symbol Formula	Vector Scalar	Unit	dimensional formula
1	**Gravitational force**	F	Vector	N	$M^1L^1T^{-2}$
2	**Universal constant of gravitation**	G	Scalar	Nm^2/kg^2	$M^{-1}L^3T^{-2}$
3	**Gravitational acceleration**	g	Scalar	m/s^2	$M^0L^1T^{-2}$
4	**Gravitational Intensity**	I	Vector	N/kg m/s^2	$M^0L^1T^{-2}$
5	**Gravitaional potential**	ϕ	Scalar	J/kg	$M^0L^2T^{-2}$
6	**Gravitational potential energy**	U	Scalar	J	$M^1L^2T^{-2}$
7	**Escape speed**	v_e	Scalar	m/s	$M^0L^1T^{-1}$
8	**Periodic time**	T	Scalar	s	$M^0L^0T^1$

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CHANGE IN THE ORBIT OF SATELLITE

Quantities	Decrease/ Increase	Relation with r
Orbital velocity	Decreases	$v \propto \frac{1}{\sqrt{r}}$
Time Period	Increases	$T \propto r^{3/2}$
Linear Momentum	Decreases	$P \propto \frac{1}{\sqrt{r}}$
Angular Momentum	Increases	$L \propto \sqrt{r}$
Kinetic Energy	Decreases	$K \propto \frac{1}{r}$
Potential Energy	Decreases	$U \propto \frac{-1}{r}$
Total Energy	Increases	$E \propto \frac{-1}{r}$
Binding Energy	Decreases	$B.\ E. \propto \frac{1}{r}$

DEPENDENCY

Physical Quantity	Dependency
➤ Gravitational force	➤ mass of both object. ➤ distance between them (r)
➤ Universal constant of gravitation (G)	➤ mass of large sphere (M) ➤ mass of small sphere (m) ➤ angle of twist in the wire in equillibrium condition. (θ) ➤ The restoring torque per unit twist = (k) ➤ length of rod = (l) ➤ distance between their centres in equillibrium condition (r).
➤ Gravitational acceleration (g_e) (on earth)	➤ mass of earth (M_e) ➤ radius of earth (R_e)
➤ Gravitational acceleration at height h.	➤ mass of earth (M_e) ➤ radius of earth (R_e) ➤ height (h) from the earth.
➤ Gravitational acceleration at depth.	➤ gravitational acceleration on the surface. ➤ depth (d) ➤ radius of earth (R_e)
➤ Gravitational Intensity.	➤ mass of object (m) ➤ Force on object by gravitational field(F)
➤ Gravitational potential (on the surface of earth)	➤ mass of earth (M_e) ➤ radius of earth (R_e)
➤ Gravitational potential energy (object of mass m) $U = -\frac{GM_e m}{R}$	➤ mass of earth (M_e) ➤ mass of object (m) ➤ radius of earth (R_e)
➤ Escape energy = $\frac{GM_e m}{R_e}$	➤ mass of earth (M_e) ➤ mass of object (m) ➤ radius of earth (R_e)
➤ total energy satellite $E = -\frac{GM_e m}{2r}$	➤ mass of earth (M_e) ➤ mass of object (m) ➤ distance of satellite from centre of earth

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Physical Quantity	Dependency
➤ Periodic time of satellite $T^2 = \left(\frac{4\pi^2}{GM_e}\right) r^3$	➤ mass of earth (M_e) ➤ orbital radius (r)

NUMERICAL INFORMATION

- Magnitude of G : 6.67×10^{-11} Nm^2 / kg^2
- The radius of earth at equator is nearly 21 *km* more than the radius at the poles.
- Gravitational acceleration on surface of earth is 9.8 m/s^2.
- Mass of earth $M_e = 6 \times 10^{24}$ *kg*
- Radius of earth R_e = 6400 *km*
- Gravitational acceleration on centre of earth is zero. ($\because$ $r = 0$)
- Gravitational potential on surface earth $\phi = -0.63 \times 10^8$ J / *kg*
- Gravitational potential at infinite distance from centre of earth = 0
- Escape speed / velocity on surfacae of earth, v_e = 11.2 *km/s*
- Escape velocity on the surface of moon v_e = 2.3 *km/s.*
  Which is nearly (1 / 5) times the escape speed of the earth's surface.
- Velocity of light c = 3×10^8 *m/s*
- The periodic time of moon's revolution around the earth is 27.3 days.
- Height of geo-stationary satellite h = 35860 *km*.
- Polar satellites are at height 800 *kg*.
- Time period of polar satellite is nearly 100 min.

Value of T^2/a^3 for some planets (For information only)

Planet	a m	T *year*	T^2/a^3 $year^2/m^3$
Mercury	5.79×10^{10}	0.24	2.95×10^{-34}
Earth	15×10^{10}	1.0	2.96×10^{-34}
Mars	22.8×10^{10}	1.88	2.98×10^{-34}
Saturn	143×10^{10}	29.5	2.98×10^{-34}

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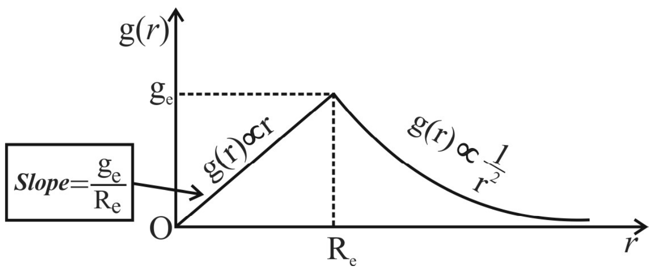
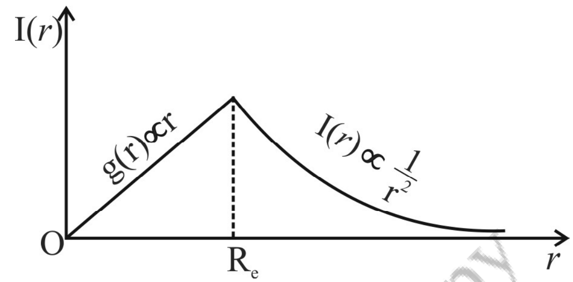
GRAPHS

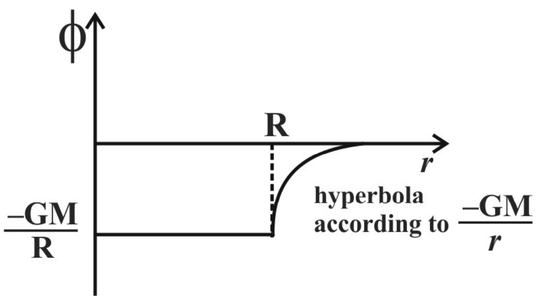
➤ **The graph of g (*r*) → *r*   *(solid sphere)***



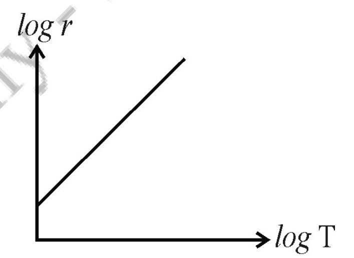
➤ **The graph of I → *r*   *(solid sphere)***



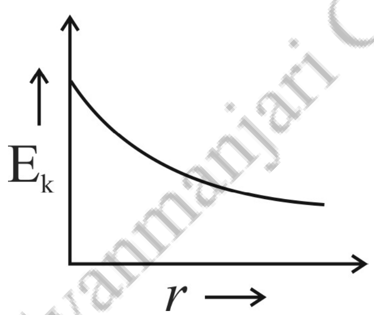
➤ **The graph of ϕ → *r*   *(hollow sphere)***



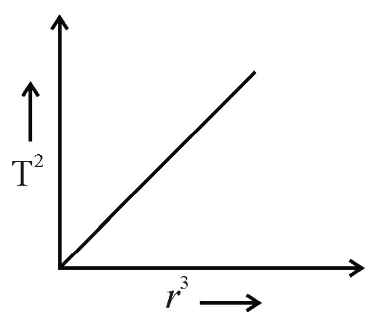
➤ **The graph of *log r* → *log T* of different satellites around a planet which has orbital radius r and the corresponding periodic time T.**



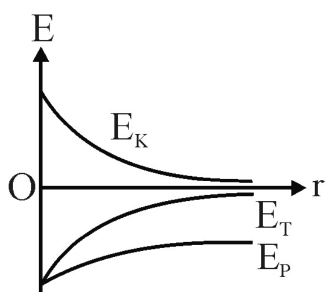
➤ **The graph of the kinetic energy of a satellite E_K and its orbital radius r.**



➤ **The graph of $T^2 \propto r^3$.**



➤ **Potenital energy, Kinetic Energy, total energy → Distance (r)**



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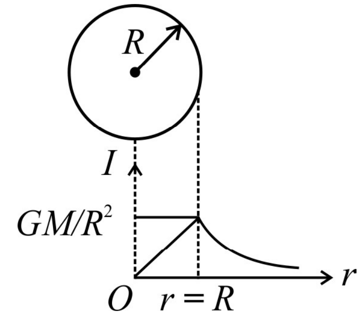
Gravitational Potential for Different Bodies

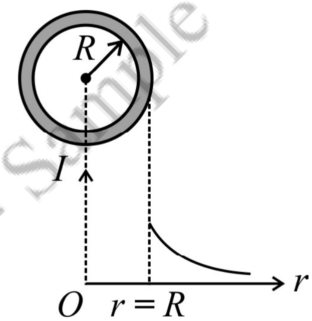
(1) Intensity due to uniform solid sphere

Outside the surface r > R	On the surface r = R	Inside the surface r < R
$I = \frac{GM}{r^2}$	$I = \frac{GM}{R^2}$	$I = \frac{GMr}{R^3}$



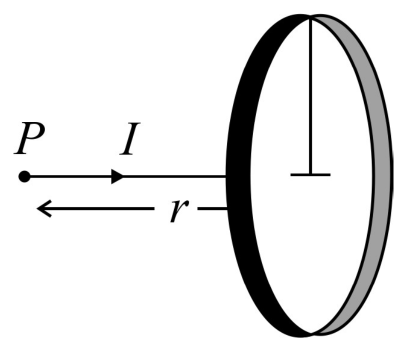
(2) Intensity due to spherical shell

Outside the surface r > R	On the surface r = R	Inside the surface r < R
$I = \frac{GM}{r^2}$	$I = \frac{GM}{R^2}$	$I = 0$



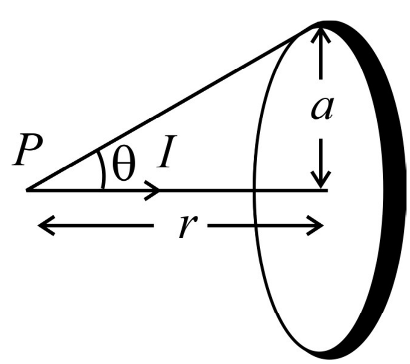
(3) Intensity due to uniform circular ring

At a point on its axis	At the centre of the ring
$I = \frac{GMr}{(a^2 + r^2)^{3/2}}$	$I = 0$



(4) Intensity due to uniform disc

At a point on its axis	At the centre of the disc
$\frac{2GMr}{a^2}\left[\frac{1}{r} - \frac{1}{\sqrt{r^2 + a^2}}\right]$ $\frac{2GM}{a^2}$	$I = 0$



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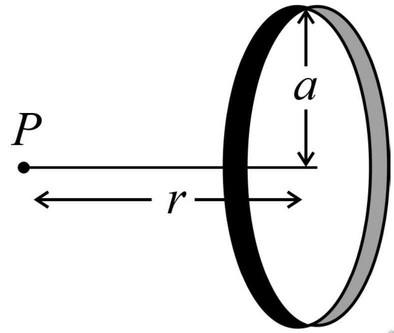
Gravitational Potential for Different Bodies

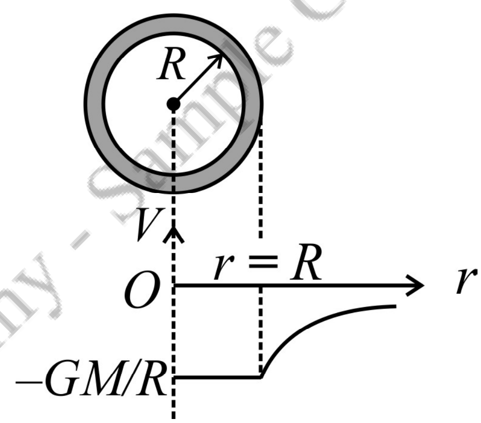
(1) Potential due to uniform ring

At a point on its axis	At the centre of the ring
$V = -\dfrac{GM}{\sqrt{a^2 + r^2}}$	$V = -\dfrac{GM}{a}$



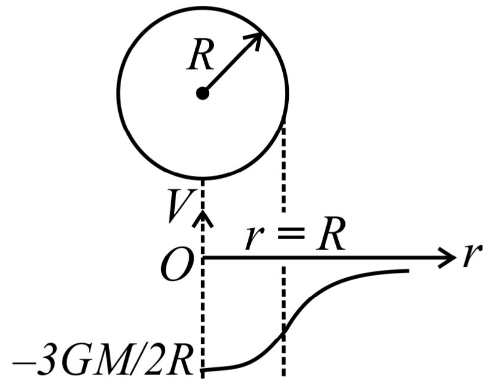
(2) Potential due to spherical shell

Outside the surface r > R	On the surface r = R	Inside the surface r < R
$\dfrac{-GM}{r}$	$V = \dfrac{-GM}{R}$	$V = \dfrac{-GM}{R}$



(3) Potential due to uniform solid sphere

Outside the surface r > R	On the surface r = R	Inside the surface r < R
$\dfrac{-GM}{r}$	$V_{surface} = \dfrac{-GM}{R}$	$V = \dfrac{-GM}{2R}\left[3 - \left(\dfrac{r}{R}\right)^2\right]$ at the centre (r = 0) $V_{centre} = \dfrac{-3}{2}\dfrac{GM}{R}$ (max.) $V_{centre} = \dfrac{3}{2}V_{surface}$



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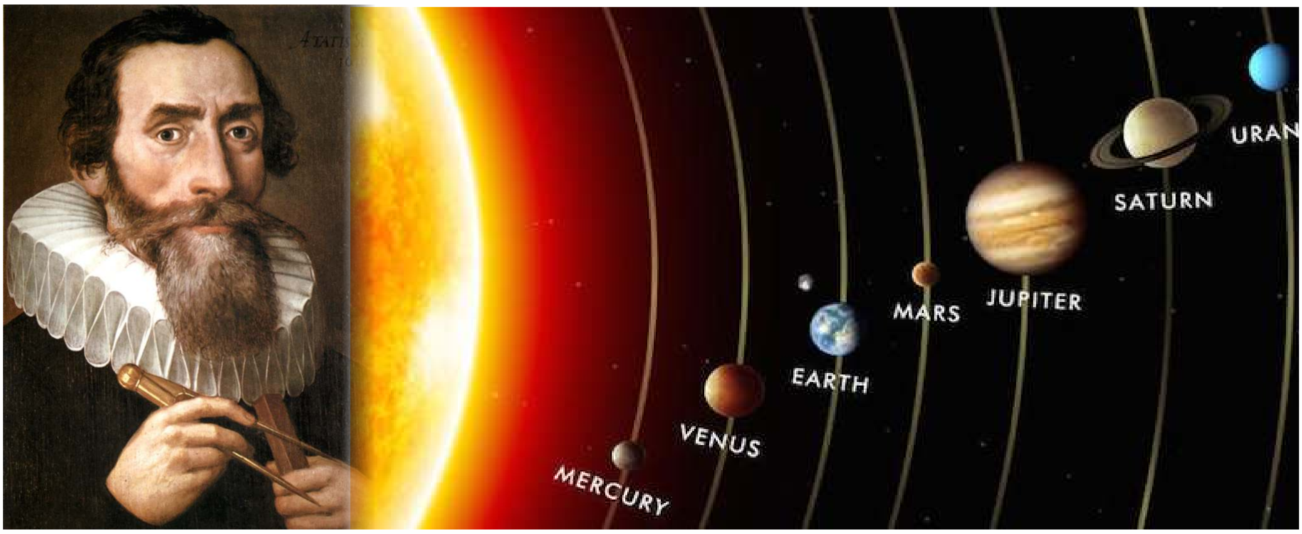
Johannes Kepler

(December 27, 1571 November 15, 1630)

Johannes Kepler was a German mathematician, astronomer and astrologer. A key figure in the 17th century scientific revolution, he is best know for his eponymous laws of planetary motion, codified by later astronomers, based on his works Astronomia nova, Harmonices Mundi, and Epitome of Copernican Astronomy. These works also provided one of the foundations for Isaac Newton's theory of Universal gravitation.

During his career, Kepler was a mathematics teacher at a seminary school in Graz, Austria, where he became an associate of Prince Hans Ulrich von Eggenberg. Later he became an assistant to astronomer Tycho Brahe, and eventually the inperial mathematician to Emperor Rudolf II and his two successors Matthias and Ferdinand II. He was also a mathematics teacher in Linz, Austria, and an advisor to General Wallenstein. Additionally, he did fundamental work in the field of optics, invented an improved version of the refracting telescope (the Keplerian Telescope), and mentioned the telescopic discoveries of his contemporary Galileo Galilei.

Kepler lived in an era when there was no clear distinction between astronomy and astrology, but there was a strong division between astronomy (a branch of mathematics within the liberal arts) and physics (a branch of natural philosophy). Kepler also incorporated religious arguments and reasoning into his work, motivated by the religious conviction and belief that God and created the world according to an intelligible plan that is accessible through the natural light of reason. Kepler described his new astronomy as "celestial physics", as "an excursion into Aristotle's Metaphysics", and as "a supplement to Aristotle's On the Heavens" transforming the ancient tradition of physical cosmology by treating astronomy as part of a universal mathematical physics.



Nicolaus Copernicus

(19 February 1473 - 24 May 1543)

Nicolaus Copernicus was a Renaissance astronomer and the first person to formulate a comprehensive heliocentric cosmology which displaced the Earth from the center of the Universe.

Copernicus' epochal book, Derevolutionibus orbium coelestium (On the Revolutions of the Celestial Spheres), published just before his death in 1543, is often regarded as the starting point of modern astronomy and he defining epiphany that began the scientific revolution. His heliocentric mode, with the Sun at the center of the universe, demonstrated that the observed motions of celestial objects can be explained without putting Earth at rest in the center of the universe. His work stimulated further scientific investigations, becoming a landmark in the history of science that is often referred to as the Copernican Revolution, Among the great polymaths of the Renaissance, Copernicus was a mathematician, astronomer, jurist with a doctorate in law, physician, quadrilingual polyglot, classics scholar, translator artist, Catholic cleric, governor, diplomat and economist.